A classical model of education, growth and distribution[1]

Amitava Krishna Dutt[2] Roberto Veneziani[3]

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[2]Department of Political Science, University of Notre Dame, Notre Dame, IN 46556, USA *and* FLACSO, Quito, Ecuador. Email: adutt@nd.edu

[3]School of Economics and Finance, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom. Email: r.veneziani@qmul.ac.uk

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Corresponding author: Roberto Veneziani, School of Economics and Finance, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom. Email: r.veneziani@qmul.ac.uk

Abstract: We develop a classical macroeconomic model to examine the growth and distributional consequences of education. Contrary to the received wisdom, we show that human capital accumulation is not necessarily growth-inducing and inequality-reducing. Expansive education policies may foster growth and reduce earning inequalities between workers, but only by transferring income from workers to capitalists. Further, the overall effect of an increase in education depends on the actual characteristics of the educational system and on the nature of labor market relations. Although the primary aim of the paper is theoretical, we argue that the model identifies some causal mechanisms that can contribute to shed light on recent stylized facts on growth, distribution and education for the US.

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1 Introduction

The role of education – and what is called human capital – has received a great deal of attention in the literature on economic growth in recent decades. The contribution of education to skill formation, and the resultant division of the labor force into high- and low-skilled workers (henceforth, H- and L-workers, respectively), has also been widely examined in the literatures on income distribution and international trade. This analysis has been conducted empirically and theoretically, and also entered popular discussions. Neoclassical growth models (including endogenous growth models) see education as promoting growth by making the productivity of labor increase more rapidly, and improving income distribution by increasing wages, although different rates of skill formation – through education – between different groups are sometimes argued to exacerbate income inequality. Surprisingly, however, education has received little or no attention in theories of growth and distribution in the classical tradition, despite the obvious relevance it has for the dynamics of capitalist economies. A recent text on growth theory written mainly from a classical perspective, by Foley and Michl (1999) has no discussion of the role of education. We are, in fact, unaware of any heterodox dynamic model of growth and distribution which analyzes the role of education.[1]

This gap is all the more surprising given the fair amount of attention devoted to education in broader political economy discussions. While much of this discussion is in relation to the ideological role of education, some of it also relates to growth. For instance, some commentators have associated the relative neglect of the education sector with the profit squeeze and the decline in productivity growth after the so-called Golden Age of Capitalism

[1]To the best of our knowledge, the only exceptions are a post-Keynesian model in Dutt (2010) and our earlier paper on classical growth theory which contains a simplified model but focuses on the contributions of the classical economists on education (Dutt and Veneziani 2011).

which ended in the late 1960s (Glyn et. al. 1990).

The purpose of this paper is to fill this gap in the literature and develop a classical model of growth and distribution in which the primary role of education is to convert L-workers into H-workers, and with which broader political-economy considerations may also be addressed.

The classical approach to *growth* has thus far focused mainly on the role of capital accumulation due to saving by capitalists (see Foley and Michl 1999, for an exposition), and due to technological change brought about by the response of capitalist firms to labor market conditions (see, e.g., Duménil and Lévy 2003; Flaschel 2009).[2] It is not obvious what effect education would have on growth and distribution according to this tradition. Given the existence of unemployed workers, increases in education, by increasing labor productivity, need not increase output and its growth. Nor, in the presence of unemployment, do they necessarily increase wages and improve income distribution.[3] Indeed, the classical framework has not been concerned with the effect of education and skill formation on *distribution*, focusing instead on the functional distribution of income between wages and profits,[4] with some attention being given to landlords, rent and natural resources in a Ricardian vein (as in Pasinetti's 1960 classic model or, more recently, in Foley and Michl 1999; and Petith 2008), to managers (Tavani and Vasudevan 2014),[5] and to financial capitalists – following Marx's

[2]There are, of course, many differences regarding the analysis of growth and distribution of those who have collectively been referred to as classical economists, such as Smith, Ricardo, Malthus, Mill and Marx, and there are also different interpretations of each classical economist. We do not discuss these exegetical issues but, rather, use a model that captures some essential characteristics common to the classical economists.

[3]Marx, for instance, argued that education would not have a positive effect on wages when the educated labor became unemployed due to the introduction of machinery because the skilled unemployed would bid down wages competing for the remaining jobs (Marx and Engels 1975-2005, Vol. VI, p. 427).

[4]See, for example, the classic analyses by Goodwin (1967) and Foley (1986). More recent contributions include Flaschel (2009).

[5]The conception and role of managerial labor is quite different from our notion of H-workers. For

distinction between interest payments and profit income (Dutt 1989). How will education and human capital affect the dynamics of distribution? Will limited access to education widen the inequality between people who obtain higher levels of education and those who do not? What are the broader, political-economy effects of education?

The main purpose of our exercise is theoretical and the paper attempts to fill a gap in the literature on classical models. Yet our motivation is not purely theoretical or exegetical. We believe that models in the classical tradition may contribute to explain some stylized facts of the dynamics of growth and distribution in advanced economies, which are at least prima facie inconsistent with the standard approach. In particular, although our analysis does not aim to provide a complete explanation of the recent dynamics of growth and inequality in the US economy, the complete model in section 5 does shed some light on important causal mechanisms that provide a counterpoint to the standard emphasis on the beneficial effects of education and human capital accumulation.

1.1 Stylized facts

To be specific, our analysis is motivated by some features of the US economy in the past four decades, which have been characterized by the rise of the knowledge economy and the predominance of neoliberal policies.

Consider distribution first: the income shares of capital and labor have remained roughly constant from 1975 to 2011, with the share of wages in national income averaging 0.64 with a standard deviation of 0.011 and the share of corporate business profits before tax averaging 0.076 with a standard deviation of 0.02 (Federal Reserve Economic Data, Federal Reserve Bank of St Louis). At the same time, earning inequalities within the labor force

managers perform purely supervisory work and their prerogatives are not linked to skills or education.

have significantly increased: the ratio of the hourly wage of College-educated workers to the hourly wage of workers with just a high school degree has moved from around 1.43 in the mid-1970s to 1.76 in 2007 (The State of Working America 2008-10, Table 3.15). Similarly, the income share of the top decile of the income distribution has moved from around 32% in the mid-1970s to 46% in 2010, with most of the gains actually accruing to the top 1% (The State of Working America 12th Edition, Figure 2AA).

Concerning growth, two features of the dynamics of the US economy stand out. The growth rate in per capita GDP has averaged 1.78% during 1975-2013 (World Development Indicators, World Bank). This is much lower than the average growth rate in the previous years characterized by the so-called Keynesian compromise. Also, and perhaps more importantly, despite significant fluctuations, there is no sign of an acceleration in growth in the last four decades. If anything, the growth rate displays a slightly declining trend.

Trends in education are equally clear, showing an increasingly educated workforce, and a seemingly more open education system that is more accessible, for example, to women and minorities. Total fall enrolment in higher education has seen a continuous and marked upward trend from 11.1 million in 1975 to 21 million in 2010, with an acceleration since 1990. At the same time, the percentage of women enrolled saw a remarkable increase from 46% in 1975 to 57% in 2010.[6] Similarly, the percentage of high school completers going to College has increased significantly across the whole population but particularly marked upward trends can be seen in the black and hispanic populations with both groups moving

[6]Source: U.S. Department of Education, National Center for Education Statistics, Opening Fall Enrollment in Higher Education, 1965; Higher Education General Information Survey (HEGIS), "Fall Enrollment in Institutions of Higher Education" surveys, 1966 through 1985; Integrated Postsecondary Education Data System (IPEDS), "Fall Enrollment Survey" (IPEDS-EF:86-99); and IPEDS Spring 2001 through Spring 2011, Enrollment component.

from around 45% in 1972 to, respectively, 69.5% and 59.3% in 2009.

The developments in education in the last four decades seem at odds with the stylized facts on growth and inequality. Based on endogenous growth theory, a significant increase in human capital should lead to a higher growth rate of the economy, ceteris paribus. Similarly, the increase in the supply of H-labor, and a wider access to education, should reduce skill differentials and have equalizing effects, ceteris paribus. The standard way to reconcile these stylized facts is precisely to deny the empirical relevance of the ceteris paribus clause. In particular, the marked increase in earning inequalities is explained by skill-biased technical change which has led the demand for educated workers to increase even more sharply than supply, thereby raising the skill differential.[7]

This is not the place for a thorough discussion of the vast literature on earning inequalities, but it is worth noting that the empirical evidence on the skill-biased technical change hypothesis is inconclusive. Standard human capital variables such as experience and education usually explain between one eighth and one third of earning inequalities, and a large part of earning inequalities occur within groups of equally educated and experienced workers – even controlling for other observable individual characteristics (see Card and Lemieux 1996; Lemieux 2006). To be sure, one may argue that these results are due to data limitations, and any residual variance in earning inequality can be attributed to unobservable individual skills. Yet Egger and Grossman (2004) have shown that cognitive skills remain of limited value to explain the variance of wage differentials among workers with the same level

[7] "In the absence of substantial skill bias in technology, the large increase in the supply of skilled workers would have depressed the skill premium" (Acemoglu 2002b, p.10). Skill-biased technical change could be either exogenous (Katz and Murphy 1992; Autor et al. 1998), or endogenously generated by the increase in the relative supply of H-labor, as in the so-called 'market size effect' (Acemoglu 2002a, 2002b).

of education.[8] More generally, there is growing empirical evidence suggesting that supply-and-demand factors alone cannot explain observed variations in wage inequality in advanced economies, and institutions, government policies, and the distribution of power among key actors in the labor market have a central explanatory role (e.g. Blau and Kahn 1996 and the contributions in Freeman and Katz 1996).[9]

From this perspective, given the presence of significant imperfections in both product and labor markets, one important question concerns the factors influencing wage setting procedures and institutions, including collective organizations, bargaining power, social norms, expectations, and so on. In particular, in a general bargaining-theoretic framework, one striking – and well-known – empirical fact is the marked decline in all indicators of workers' bargaining power and cohesion. Union density remained reasonably stable during 1947-1979 with an average of 24.45% (and a standard deviation of 0.02) but has since steadily declined reaching a minimum of 11.30% in 2003. The drop has been even more marked in the private non-agricultural sector where unions have virtually disappeared and the percentage of workers covered by a collective agreement has moved from 23.6% in 1979 to 7.4% in 2012 (Hirsch and Macpherson 2003 and updates). A large number of empirical studies have proved that unionization is one of the key variables in explaining income and earning distributions.

Therefore, although directed technical change is a key determinant of the dynamics of growth and distribution, and competitive mechanisms and optimal choices play a funda-

[8]Focusing on a very detailed dataset collected by the OECD, Franzini and Raitano (2014) have proved that wage differentials remain largely unexplained even including individual information about the details of educational paths, employment history, and other acquired skills (e.g. knowledge of IT).

[9]Empirical studies in different countries indicate that the rapid increase in educational levels in the labor force may have actually been faster than the educational requirements of jobs, leading to tendencies toward overqualification among those employed (Borghans and de Grip 2000).

mental role in determining skill differentials, we explore the connections between education, growth and distribution in a classical model where institutions, bargaining, and social norms are equally important.

The rest of this paper proceeds as follows. Section 2 discusses the theoretical foundations of our classical model of education, growth and distribution. Section 3 lays out the benchmark model. Section 4 examines the dynamics of the economy with given bargaining conditions. Section 5 generalizes the analysis of the growth and distributive implications of education to economies in which the workers' bargaining power is endogenously determined. Section 6 relates our model and results to the stylized facts. Section 7 concludes.

2 Education in a classical framework

This section discusses how this paper incorporates education in a classical framework, by considering three key questions: how is a classical economy characterized? What exactly is the role of education in the economy? What determines changes in the level of education? We examine each question in order to justify our modelling strategy.

2.1 Characteristics of the economy

We characterize the classical economy as having five key features. First, there is limited substitution between capital and labor, the two basic factors of production, so that we can simplify by assuming fixed coefficients for labor and capital in production.[10] In addition to

[10]For many classical models this is not a crucial assumption because distribution is assumed to be exogenously fixed (see below) and so profit-maximizing behavior generally implies a particular chosen technique even with capital-labor substitution. When one allows changes in distribution, however, inelastic factor substitution plays a more important role.

capturing some key tenets of the classical tradition, this assumption is in line with recent empirical evidence casting doubts on the substitutability between labor and capital, and on marginal productivity theory as an explanation of the distribution of income between capital and labor (Foley and Michl 1999; Basu 2009).

Second, there are two *basic* classes, capitalists and workers, with capitalists owning capital and receiving income only from profits, and workers receiving income as wage labor only.

Third, capitalists save and workers do not. The assumption that workers do not save is, of course, a simplification (the key hypothesis is actually that they save at a lower rate than capitalists). Following Kaldor (1966), one may argue that differential saving rates are rooted in the structure of a corporate economy: the saving rate out of profits is high because firms retain a fraction of their profits.

But our assumption can also be seen as a stark way of capturing the extreme inequalities in wealth distribution, and the relatively low degree of social mobility, that characterize capitalist economies.[11] Although both in our model and in actual economies, there are relevant cleavages, and earning inequalities, *within* the working class, we think that our assumption reflects – at least as a first approximation – the fact that the vast majority of wage laborers have zero net wealth (Allegretto 2011; Piketty 2014; Krueger et al 2016).[12] It

[11]To be sure, the degree of social mobility in advanced economies is not zero. Nonetheless, recent research suggests that it is remarkably (and perhaps surprisingly) low and declining (Greenstone et al 2013). This is true even if one considers the very long run (Barone and Mocetti 2016) and without major differences between European countries and the US (Bénabou and Tirole 2006, pp.702ff). For a formal analysis of income inequality and mobility in economies with heterogeneous skills, see for example Garcìa-Peñalosa and Turnovsky (2015) and Alonso-Carrera et al (in press).

[12]It is perhaps worth stressing again that H-workers in our model are neither managers, nor supervisors. They are workers employed either in actual productive activities or in the education sector.

is also in line with recent findings by Krueger et al (2016) who report higher consumption rates for agents with lower net worth.

The main reason to rule out workers' savings is, however, theoretical: we abstract from saving by workers in order to abstract from the dynamics of worker-owned capital (Foley and Michl 1999; Dutt and Veneziani in press) and focus on the effect of education in a starkly class-divided economy. We shall return to this issue in the concluding section.

Fourth, the economy has unemployed workers, and the distribution of income is affected by the relative bargaining power between capitalist firms and workers.

Finally, all savings are invested. This implies that the economy has no aggregate demand problems, because there is investment demand from the entire income that is not consumed. While this is an appropriate assumption for Smith and Ricardo, who both assumed versions of Say's Law, it is not faithful to either Malthus, who emphasized the problems of over-saving and gluts, or Marx, who clearly recognized that capitalist producers can hoard rather than invest if they have low profit expectations, and this can result in what Marx called a realization problem and reduce output. However, Marx seems not to have stressed these problems in analyzing long-run growth. Without taking a view on such exegetical issues, we ignore effective demand problems and assume that all savings is automatically invested consistently with almost all growth models in the classical-Marxian tradition.[13]

The assumptions on savings behavior and on effective demand imply that an increase in the share of income going to profits has a positive effect on the growth rate of the economy.

[13]See, e.g., Harris (1978), Marglin (1984), Dutt (1990) and Foley and Michl (1999). Even when aggregate demand is allowed to determine output in the short run, as in Duménil and Lévy (1999), in the long run the economy is assumed to produce at the potential level of output with saving equal to investment. Dutt (2011) provides a thorough discussion of the role of aggregate demand in the writings of the classical economists and modern classical-Marxian growth theory.

Although disputes on whether advanced economies are indeed *profit-led* (rather than *wage-led*) are far from settled, there is a fair amount of evidence suggesting that so-called liberal market economies in which earning inequalities and skill differentials have increased most dramatically, such as the US and the UK, are profit-led, consistent with our model (Barbosa-Filho and Taylor 2006). There is also evidence that advanced economies have become more profit-led during the last decades of neoliberal policies (Onaram et al 2011; Carvalho and Rezai 2016).

2.2 Role of education in the economy

Education plays a complex and multifaceted role in the economy, and we try to capture some important aspects of it. First, education has a direct role in determining the dynamics of the stock of human capital and knowledge by transforming L-workers into H-workers. The main difference between the two types, in terms of their functions in the economy, is that L-workers are simply an input into the production of the final good, while H-workers have a more complex role. They also serve as a – distinct – input in the production of the final good. In addition, and more importantly, H-workers have a number of other functions: they increase the efficiency of both L- and H-workers through the process of innovation, and they also help in the process of education, as mentors or educators. While L-workers tend to be employed in routine production activities, H-workers are innovators. Unger (2007, pp.96-97), who distinguishes such roles in terms of ideas about the mind, expresses it as follows:

> We know how to repeat some of our activities, and we do not know how to repeat others. As soon as we learn how to repeat an activity we can express our insight in a formula and embody the formula in a machine ... The not yet repeatable part of our activities – the part for which we lack formulas and

> therefore also machines – is the realm of innovation, the front line of production. In this realm, production and discovery become much the same thing.

We therefore assume that education converts workers who could only do repetitive tasks into discoverers and innovators, although they continue to be engaged in some routine production activities that are, however, qualitatively different from those of L-workers.

This formulation of the role of education can be found in the writings of classical economists, including Smith (1776, p.282) and McCulloch (1825, p.122). McCulloch, in fact, emphasized – much more than his contemporaries – the role of education and the diffusion of knowledge in increasing growth through technological change (see O'Brien 1975, p.217).[14] It is now also fairly common in neoclassical growth theory, which stresses that education and the accumulation of human capital increase productivity growth. However, there are differences between the neoclassical approach and ours.

The standard view in neoclassical growth models with education is that there is no essential difference between workers who are educated and those who are not (e.g., Uzawa 1965; Lucas 1988). Unskilled labor accumulates human capital through education and becomes skilled labor, with a higher productive power. One worker becomes more than a worker in efficiency units. In this approach, workers are qualitatively the same, and can be shifted between the educational system and actual goods production. This is in contrast to our model, in which education converts L-workers into H-workers who are qualitatively different.

Our approach is closer to the neoclassical trade-theoretic literature, which takes L- and H-labor to be qualitatively different and distinct inputs. Models in this vein are used to examine, for example, the implications of trade liberalization for the relative wages of skilled and unskilled workers in developed countries (e.g., Stokey 1991; Wood 1994).

[14] See O'Donnell (1985) for a review of classical ideas on education.

In our model, however, in addition to being qualitatively different *inputs* used in production, H- and L-workers also have different *roles* in the economy. In this respect our approach shares some characteristics with some other models with different types of labor. Galor and Zeira (1993), for example, consider two types of technologies, one which uses capital and skilled workers and the other which uses no capital and unskilled labor; thus skills enable workers to work with capital while unskilled workers cannot do so. In other models the roles are different because H-workers produce differentiated intermediate goods, whereas L-workers only "assemble" the intermediate goods to produce the final good (see Dutt 2005). We combine the two approaches, allowing both L- and H-workers to be inputs into the production of the final good, but also allowing only H-workers to have a role in inducing productivity growth and in the process of education.

The economic relevance of education, however, does not reduce to its influence on the dynamics of human capital and technological progress. Authors working in the classical and Marxian tradition have long emphasized the multidimensional role of education in influencing the growth and distributive outcomes of capitalist economies. This literature incorporates broad issues such as the role of education in determining social outcomes as the product of conflict and/or cooperation. On the one hand, Marxist and radical scholars have emphasized the role of education in weakening the position of workers by dividing them into groups based on their level of education, in creating and strengthening the perception of upward socio-economic mobility and thereby increasing tolerance for income inequality, indoctrination and socialization, and easing the process of the extraction of effort (and hence labor productivity and profits) (see, among many others, Giddens 1973; Bowles and Gintis 1975, 1976).

On the other hand, some classical economists viewed education in a more positive light. Education, McCulloch (1825, p.134) believed, would show workers "how closely their inter-

ests are identified with those of their employers, and with the preservation of tranquility and good order". Smith (1776, p.782) argued that education could play a key role in creating a more informed and discerning polity by contrasting the tendency of the increasing division of labor to make workers "as stupid and ignorant as it is possible for a human creature to become". Even Marx (1867, p.453) saw education as "the *only* method of producing *fully developed human beings*", although he was thinking not of education in the form actually existing in his time but of an ideal system, arguably in a classless society.

In section 5 we explore some of these broader issues by endogenizing the bargaining power of workers and analyzing the implications of different education systems on workers' attitudes, norms, and beliefs.

2.3 Dynamics of education

In our approach the rate of change in the number of people educated depends positively on the number of educators and mentors; on the wage of H-workers relative to that of L-workers; and on the broad institutional and organizational characteristics of the education system, and in particular the access to education which captures factors such as the degree of openness of the education system and the availability of educational loans.

By assuming that the wage differential affects the dynamics of the stock of H-workers, our model is consistent with standard approaches emphasizing the relevance of individual preferences (reflected, for instance, in their rate of time preference) and the returns to schooling in the accumulation of human capital. However, it stresses other factors, such as the degree of access to education, capturing the relevance of institutions and norms. Our approach is therefore less specific than the standard one, but we consider this lack of specificity to be a virtue because it opens up space for other, oft-neglected determinants of education.

3 The benchmark model

In this section we set out the basic classical model with two kinds of labor, high- and low-skilled, the quantities employed of which are H and L, and which receive real wages w_H and w_L. We use the symbol H_P to distinguish between H-workers employed in production from those employed as educators, denoted by H_E. The ratio of skilled to unskilled wage, the *skill premium*, is

$$\sigma = w_H/w_L. \tag{1}$$

With capital, we therefore have three inputs in the economy, which correspond to three classes in society: capitalists who own physical capital, H-workers each with one unit of H-labor and L-workers each with one unit of L-labor. We examine a closed economy in which the government taxes to finance education but has no other fiscal functions.

Technology is as follows. There is only one sector producing one good which can be used both for consumption and for (capital) investment. Production uses fixed coefficients input-output relations with capital and a mixture of H- and L-labor as inputs into production. The productivity of H- and L-labor is given at a point in time by A_H and A_L, respectively, and the maximum output that can be produced by a unit of capital is k. To be specific, the production function of the standard firm is:

$$Y = \min\{kK, f(A_L L, A_H H_P)\}, \tag{2}$$

where Y is the output of the good, K is the amount of capital, and f is homogeneous of degree one, which is consistent with the fixed coefficients structure. This function is in

line with standard classical assumptions in rejecting the substitutability between labor and capital, but in principle it allows for substitutability between the two types of labor. In the rest of this paper, for the sake of analytical convenience, and without significant loss of generality, we assume a CES specification for the f function.

Assumption 1 (A1). $Y = \min\{kK, [(A_L L)^\rho + (A_H H_P)^\rho]^{\frac{1}{\rho}}\}$, with $\rho < 0$.

The CES structure is very general and is widely used in the literature on skill differentials and human capital (see Autor et al. 1998; Acemoglu 2002a, 2002b). We assume limited substitutability between the two types of labor. Although the received view is that $\rho > 0$, recent empirical studies raise doubts on this finding and suggest that elasticity of substitution is below one (Card et al. 1999; Skott and Slonimczyk 2012), a view which we believe to be more plausible. Further, a negative value of ρ makes the analytical results starker and avoids some unnecessary technicalities. However *all* of the main conclusions of the paper continue to hold if $\rho < 0.5$, which encompasses the standard estimates of the elasticity of substitution.

Next, we assume that H-workers are more productive at all t, and that their productivity advantage remains constant over time. Formally:

Assumption 2 (A2). There is a scalar $\mu \geq 1$, such that $A_H = \mu A_L$, all t.

This assumption encompasses the special case with $\mu = 1$, at all t, and allows one to analyze the effect of increases in productivity differentials on growth, distribution, and the relative composition of the labor force. The assumption that μ is constant should be taken as an approximation and a first step to a more complete analysis. Yet, this seems reasonable (if not necessary) in a steady state, such that if any loss of generality occurs, this only has to do with the analysis of the transition path.

Given A1 and A2, the optimal demands for H- and L-labor by profit-maximizing, perfectly

competitive firms (we consider one representative firm, with all firms being identical) are:[15]

$$H_P^D = \frac{kK}{\left[\left(\frac{w_H}{w_L}\right)^{-\frac{\rho}{\rho-1}} \mu^{\frac{\rho}{\rho-1}} + 1\right]^{\frac{1}{\rho}} A_L \mu} = \frac{b(\sigma)K}{A_L}, \tag{3}$$

where $b(\sigma) = k\mu^{-1}\left[\sigma^{\frac{\rho}{1-\rho}} M + 1\right]^{-\frac{1}{\rho}}$ and $M = \mu^{\frac{\rho}{\rho-1}}$, and

$$L^D = \frac{kK}{\left[1 + \left(\frac{w_H}{w_L}\right)^{\frac{\rho}{\rho-1}} \mu^{-\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}} A_L} = \frac{c(\sigma)K}{A_L}, \tag{4}$$

where $c(\sigma) = k\left[\sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1\right]^{-\frac{1}{\rho}}$. Given A1, it follows that $b' < 0$ and $c' > 0$, where a prime symbol indicates a derivative. Further, as σ tends to zero, $b(\sigma)$ tends to infinity and as σ tends to infinity, $b(\sigma)$ tends to k/μ, and the function $b(\sigma)$ is inelastic for all σ. The function $b(\sigma)$ is shown in Figure 1.

There exists a public sector responsible for delivering educational services. The government is assumed to employ a fraction $\varepsilon \in [0, 1)$ of the total supply of H-workers, H^S, which is given at a point in time, and pay them the wage for H-workers, w_H. The rest of the H-workers are available for employment in the private market.

The markets for the two kinds of workers are as follows. L-workers are in unlimited supply, and along standard classical-Marxian lines we assume that their real wage is determined by their relative bargaining power vis-à-vis firms (see, e.g., Goodwin 1967; Marglin 1984; Dutt 1990; Foley and Michl 1999; Duménil and Lévy 2003).[16] We parameterize this state in terms of the real wage of L-workers relative to their efficiency factor, A_L, so that given bargaining

[15]The full derivation of the labor demand functions can be found in the online Addendum.

[16]The assumption of an unlimited supply of L-workers, or equivalently, of a sufficiently high growth rate of the labor supply, allows us to ignore all labor constraints in the analysis of the long-run equilibria below.

conditions, an increase in A_L results in a proportionate increase in w_L.

The market for H-workers is flexprice, and the skill premium adjusts in response to the excess demand for H-workers, given the supply of these workers, denoted as $(1-\varepsilon)H^s$, and given w_L. The L-worker wage serves as a reference point, and given the skill premium, a high w_L increases w_H proportionately. Formally:

Assumption 3 (A3). There exists a given positive scalar, λ, such that $w_L = \lambda A_L$. Further, given H^s, at any t, σ solves $(1-\varepsilon)H^s = b(\sigma)K/A_L$.

Given the assumptions on the labor market, in what follows we use the symbols H and L, to denote the quantities of H- and L-workers employed in production or education. The level of λ is determined by the relative bargaining power of L-workers.

Consistent with Kaldor's (1961) famous observation, we assume that technical progress increases the productivity of labor inputs but not of capital inputs. We formalize the relationship between the use of H-labor and labor productivity by assuming that the growth rate of the productivity of H-workers depends positively on the amount of H-labor in efficiency units as a ratio of the capital stock (used as a scaling factor representing the size of the productive economy). Without significant loss of generality, we adopt a simple linear functional form, and denoting growth rates by overhats, we assume:[17]

Assumption 4 (A4). There exist two scalars τ_0 and τ_1 with $\tau_0 \geq 0$ and $\tau_1 > 0$ such that

$$\widehat{A}_H = \tau = \tau_0 + \tau_1 \frac{(A_H H)}{K}. \tag{5}$$

Several features of A4 are worth noting. First, since all firms are identical, A_H can be thought

[17]It is important to stress that *all* of our results continue to hold under more general functional forms and assuming weaker effects of knowledge on productivity.

of as the average productivity of H-workers. Thus, although there may be externalities involved, they are not required. Second, by A4, both H-workers in production and educators contribute to technological change. This is consistent with our focus on higher education, but if educators contribute to technological change only indirectly by educating L-workers, we can make technological change depend on $(1-\varepsilon)H$, which will not change our results as long as ε is constant. Third, A4 implies that technical change is not driven only by the *level* of skills: a higher skill *intensity* of the economy, measured by the stock of (effective) H-labor relative to the size of the productive economy, has a positive effect on innovation, a plausible assumption. It is worth noting, however, that we use the capital stock as a scaling factor mostly for technical reasons.[18] Finally, A4 allows for the *possibility* of exogenous growth in productivity, i.e. $\tau_0 > 0$, as a matter of generality. However, all of our results continue to hold in a pure endogenous growth framework with $\tau_0 = 0$.

Because A_H and A_L are in general different, in principle equation (5) would not be sufficient to describe the behavior of labor productivity over time. By A2, however, it follows that $\hat{A}_L = \hat{A}_H$, all t, and we can write

$$\hat{A}_L = \tau_0 + \tau_1 \mu \frac{(A_L H)}{K}. \tag{5a}$$

In other words, as in Romer's (1990) classic paper, we distinguish two types of knowledge: skills acquired via education represent knowledge embodied in human capital, which is rival and excludable. In contrast, we conceptualize innovations as non-rival, non-excludable

[18] Our results would remain unchanged if the amount of H-labor in efficiency units was normalized, for example, using aggregate output. Further, A4 could be generalized to include capital accumulation – another proxy for learning-by-doing processes – as a determinant of technical progress. This would significantly increase the amount of technicalities but it would not alter the qualitative properties of the model.

products of learning-by-doing processes and innovation activity by H-workers with an immediate spillover to L-workers, or as H-workers developing new methods of production, or new instructions for mixing together productive inputs, which also increase L-worker productivity.[19] The improved technology can be immediately adopted by all firms without investing in new equipment.

L-labor is converted into H-labor through the process of education as follows:

Assumption 5 (A5). The supply of high-skilled labor H changes over time according to

$$\frac{dH}{dt} = \theta g(\sigma)\varepsilon H, \tag{6}$$

where $\theta > 0$ is a scalar, $g : \mathbb{R}_+ \to \mathbb{R}_+$ is convex and continuously differentiable, and there exists a value $\sigma_{min} \geq 1$ such that $g(\sigma) = 0$ for all $\sigma \leq \sigma_{min}$ and g is strictly increasing for all $\sigma > \sigma_{min}$.

Thus, the change in the stock of H-workers depends, first, on the demand for education which, in turn, depends positively on the skill premium, which increases the 'return' to education. Second, it depends on the stock of educators.[20] Non-educator H-workers can

[19]Similar specifications are adopted in the standard literature on education, innovation and economic growth where technical change depends on the time spent in education (see, for example, the classic papers by Uzawa 1965 and Lucas 1988). The main difference is that we assume that *both* educators *and* H-workers contribute to technical change. Our analysis is also similar to that of Aghion and Howitt (1998) who assume that the (expected) rate of arrival of innovations depends on the level of employment in the research and development sector. Their model, however, does not take education into account.

[20]Because every agent is endowed with one unit of time, the equation of motion of human capital could be equivalently interpreted as taking domain in the *fraction of time* the H-workers spend educating and mentoring others, rather than the *number* of educators. However, the latter interpretation is more natural given the existence of a public sector responsible for delivering educational services.

also positively affect the change in the stock of H-workers by increasing the support for, and access to, education (for instance, a higher stock implies a higher number added from H-worker families), but we do not explicitly model this; it does not affect our results qualitatively. Third, it depends on a parameter, θ, which captures the broad institutional and organizational characteristics of the education system, and in particular its degree of openness, either through government policy or through the degree of exclusivity of the higher education system and also, indirectly, the functioning of credit markets, in their role of financing education.[21] Easier access to low-cost public higher education and to student loans and grants, a more open private higher education system which is less elitist on the basis of class and income, and an improved *basic* education system for all workers which makes L-workers better prepared for higher education would all increase θ.[22]

To be sure, many different factors determine the influence of the education system (and more generally the transmission of knowledge in a society) on the creation of skills. We regard the parameter θ as a parsimonious way of modeling such influences, and thus potentially the role of public policy in the creation of skills. It can be seen as a black box, capturing the multifaceted influence of education on the dynamics of human capital.

We do not want to unnecessarily restrict our analysis and, apart from some mild regularity

[21]The parameter θ may also be seen as capturing the productivity of each educator, where productivity is interpreted in a broad sense to encompass the institutional and organizational characteristics of the education system. Given our focus on education policies, however, we consider changes in θ due to changes in the openness of the education system, rather than – for example – in educational technology per se. We are grateful to an anonymous referee for suggesting this alternative interpretation.

[22]We assume $\theta > 0$ consistent with our focus on developed economies, but we could generalize the model to allow the parameter θ to be zero or even negative. This would correspond to a backward economy, or to a dysfunctional education system in which knowledge and skills are not transmitted, and the stock of human capital is stationary or even decreasing.

conditions, do not specify an explicit functional form for g. The only theoretical restriction concerns the definition of σ_{min}: A5 incorporates the intuition that no one seeks education if the wage premium falls below a certain level. This seems rather reasonable at a theoretical and empirical level.[23]

We make the following assumption about consumption and saving behavior.[24]

Assumption 6 (A6). Workers – both high- and low-skilled – do not save, but consume their entire income; capitalists save a fixed fraction, s, of their profits.

The pre-tax income of profit recipients, or capitalists, is given by

$$Y_C = Y - w_L L - w_H(1-\varepsilon)H.$$

We assume that the government finances its educational expenditure, devoted entirely to the payment of wages of educators (abstracting from non-wage costs for simplicity), by taxing profits. This is consistent with A6 and it is broadly in line with the historical analysis of education in class-divided societies in Galor and Moav (2006). However, the assumption is made mostly for simplicity and without loss of generality.[25] Thus, if T_p denotes the aggregate taxes on profits, the *net* income of capitalists is

[23]This assumption can also be related to Smith's (1776, pp.118-19) explanation of the "difference between the wages of skilled labour and those of common labour" and to Mill's (1848, pp.386-7) analysis of the relation between skills and wages.

[24]Given A6, becoming skilled does not allow workers to start saving and accumulating wealth. The incentive to acquire skill derives from the higher wage, and thus the higher consumption standard, that one can obtain relative to remaining unskilled, as captured by the parameter σ_{min} in A5.

[25]The model could be extended to include taxes on wages although at the cost of a substantial increase in technicalities.

$$rK = Y_C - T_p = Y - w_L L - w_H(1-\varepsilon)H - T_p,$$

where r is the profit rate net of taxes. We assume that the government keeps a balanced budget, so that $T_p = w_H \varepsilon H$, and the latter expression can be written as

$$rK = Y_C - w_H \varepsilon H = Y - w_L L - w_H H. \tag{7}$$

Hence, total consumption expenditure is

$$C = (1-s)rK + w_L L + w_H H. \tag{8}$$

This implies that saving is given by the standard equation,[26]

$$S = srK. \tag{9}$$

Finally, regarding investment, we have the following.

Assumption 7 (A7). Saving and investment, I, are identically equal.

Capitalists save in order to invest, so that saving and investment are always equal. This version of Say's law is a standard assumption of models in the classical tradition, as discussed earlier. Equation (9) and A7 imply

$$I = srK. \tag{10}$$

Further, there is no effective demand problem, so that, given the existence of unemployed

[26]Observe that since government revenues are used to pay educators, and given the balanced budget assumption, profit taxation has no effect on aggregate consumption and savings.

L-workers, we have

$$Y = kK. \tag{11}$$

This macroeconomic condition justifies the microeconomic profit-maximizing decision made by each firm to produce at full capacity, as noted earlier.

We examine the determinants of growth and distribution focusing on several different variables. The growth rate of the economy can be conveniently measured by $\hat{K}$, which is equal to $\hat{Y}$. Since we do not track the overall supply of labor (which may be taken to be growing at an exogenous rate), $\hat{K}$ also proxies the growth rate of *per capita* output.

Distribution can be measured by the income shares of the three classes: $\lambda c(\sigma)/k$ for L-workers, $\sigma\lambda b(\sigma)/k(1-\varepsilon)$ for H-workers, and r/k for capitalists. The distribution of income between different workers can be measured by $\nu = w_H H/w_L L$, or by equations (2)-(4)

$$\nu = \frac{\sigma^{-\frac{\rho}{1-\rho}}\ \mu^{\frac{\rho}{1-\rho}}}{1-\varepsilon}$$

which is increasing in σ. Thus intra-workers inequalities can be equivalently analyzed either in terms of ν or in terms of σ.

We also examine the well-being of H- and L-workers by focusing on the growth rates of their wages, $\hat{w}_H$ and $\hat{w}_L$. Further, given our assumptions on the supply of L-labor, we cannot explicitly analyze the dynamics of the *unemployment* rate. Hence, we capture changes in workers' welfare by focusing directly on the growth rates of *employment*, $\hat{H}$ and $\hat{L}$.

Another variable of interest is the skill composition of employed workers (a proxy of the skill composition of the labor force), which is given by the ratio $H/L = b(\sigma)/c(\sigma)(1-\varepsilon)$, which is strictly decreasing in σ.

4 Education, growth and distribution with constant bargaining conditions

We examine the dynamics of the economy by considering two runs, for now assuming that λ, the distributional parameter, is exogenously given. In the short run, K, H and A_L are fixed, and the model solves for Y, L, σ, r and I from equations (3), (4), (7), (10), and (11). The profit rate is given by

$$r = k - \frac{w_L c(\sigma)}{A_L} - \sigma \frac{w_L b(\sigma)}{A_L(1-\varepsilon)} \tag{12}$$

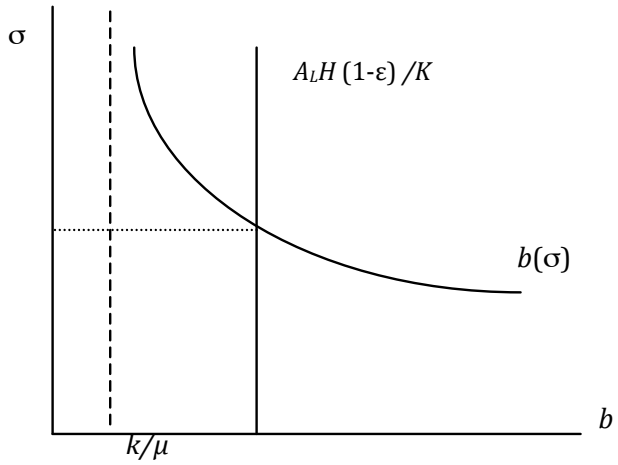

Figure 1: Determination of the skill premium in the short run.

Let $h = A_L H/K$. The short-run equilibrium value of σ is found as shown in Figure 1:

$$\sigma = \sigma(h) \equiv b^{-1}(A_L H(1-\varepsilon)/K) = \mu \left[\left(\frac{k}{\mu h(1-\varepsilon)} \right)^{\rho} - 1 \right]^{\frac{1-\rho}{\rho}}. \tag{13}$$

By A1 $\sigma(h)$ is strictly decreasing and strictly convex for all $h \in (k/\mu(1-\varepsilon), \infty)$.

In the long run K, H and A_L can change. Assuming away the depreciation of capital

without loss of generality, the change in capital stock is given by

$$dK/dt = I, \tag{14}$$

and changes in H and A_L are governed by equations (6) and (5a).

We examine the time path of the economy in the long run by focusing on the state variable $h = A_L H/K$. Because the growth rate of h is given by

$$\hat{h} = \hat{A}_L + \hat{H} - \hat{K}, \tag{15}$$

we can substitute from equations (5a), (6), (10) and (12) through (14), to obtain

$$\hat{h} = \tau_0 + \tau_1 \mu h + \theta \varepsilon g(\sigma(h)) - s\left[k - \lambda c(\sigma(h)) - \sigma(h)\lambda h\right]. \tag{16}$$

The economy is defined as being in long-run equilibrium when $\hat{h} = 0$. Proposition 1 proves the existence of multiple long-run equilibria.

Proposition 1 *Assume A1-A7. Suppose that*

$$\tau_0 + \frac{\tau_1 k}{1-\varepsilon}\left[M+1\right]^{-\frac{1}{\rho}} < sk\left[1 - \lambda\mu^{-1}\left[M+1\right]^{-\frac{1}{\rho}}\left[M + \frac{1}{1-\varepsilon}\right]\right]. \tag{KTB}$$

Then there are two long-run equilibria: E_1 with $\sigma_1 > 1$ and E_2 with $\sigma_2 < 1$. E_1 is dynamically stable, whereas E_2 is dynamically unstable.

Proof. By equation (16), it follows that

$$\frac{d\hat{h}}{dh} = \tau_1\mu + \varepsilon\theta g'(\sigma)\sigma'(h) + s\lambda\sigma(h)\left[\varepsilon - (1-\rho)\frac{\left(\frac{k}{\mu(1-\varepsilon)h}\right)^{\rho}}{\left(\frac{k}{\mu(1-\varepsilon)h}\right)^{\rho} - 1}\right].$$

Therefore, by A1 as $h \to k/\mu(1-\varepsilon)$, and σ tends to infinity, then $\hat{h} \to \infty$ and $\frac{d\hat{h}}{dh} \to -\infty$, whereas as $h \to \infty$, and σ tends to zero, then $\hat{h} \to \infty$ and $\frac{d\hat{h}}{dh} \to \tau_1\mu$. If the condition in the antecedent of the proposition is true, this implies that at $h = b(1)\left(1-\varepsilon\right)^{-1} = k\mu^{-1}\left[M+1\right]^{-\frac{1}{\rho}}\left(1-\varepsilon\right)^{-1}$, $\hat{h} < 0$, and thus by continuity there are at least two points $h_1 = b(\sigma_1)(1-\varepsilon)^{-1}$ and $h_2 = b(\sigma_2)(1-\varepsilon)^{-1}$, where $h_2 > h_1$, $\sigma_1 > 1$ and $\sigma_2 < 1$, such that $\hat{h} = 0$. Furthermore, by A1 and A5, it follows that $\hat{h}$ is strictly convex and so there exist exactly two such points. Stability follows in the usual manner noting that $\hat{h} > 0$ for all $h < h_1$ and $h > h_2$, whereas $\hat{h} < 0$ for all $h_1 < h < h_2$. ■

Although Proposition 1 identifies two equilibria, only one of them, E_1, is economically meaningful because it is dynamically stable and has a skill premium greater than one. Therefore in what follows we focus on E_1.[27]

Condition (KTB) is sufficient (but not necessary) for the existence of a long-run equilibrium and it depends on the conditions of capital accumulation (K), technical change (T) and bargaining (B). It is more likely to hold the lower τ_0, τ_1, λ, and the higher s, k. If technical progress is too strong, or if profits and capital accumulation are too slow, then the dynamics of innovation may dominate and lead the economy onto an explosive path.[28]

[27]We note in passing that at E_1, Kaldor's (1961) celebrated stylized facts are verified: constant factor shares, increasing labor productivity, and a positive rate of capital accumulation.

[28]Along this explosive path eventually H no longer increases, but h keeps increasing as technological change occurs faster than capital accumulation. This kind of self-reinforcing dynamics of technological innovation with a constant stock of education, however, is unlikely to occur in practice, and the τ function

Proposition 2 is the main result of this section. It analyzes the effect of education policies on growth, employment, and distribution.[29]

Proposition 2 ***(Education, growth and distribution)***. *Assume A1-A7. Suppose that (KTB) holds and* $\sigma_1 > \sigma_{min}$. *At* E_1, *an increase either in* θ *or in* ε *implies that in equilibrium the income share of L-workers increases relative to that of H-workers, but so does the profit share in national income. Furthermore, the skill composition of employed labor, the growth rate of the economy, and the growth rate of technical progress all increase. Finally, there exists a* $\sigma^* > 0$ *such that if* $\sigma_1 > \sigma^*$ *then a sufficiently small increase either in* θ *or in* ε *yields an increase in the growth rate of human capital and employment.*

Proof. 1. Consider θ first. Because $\sigma_1 > \sigma_{min}$, by equation (16) it follows that an increase in θ yields an upward shift of the $\hat{h}$ schedule. Since (KTB) holds, then there exists a long run equilibrium at $\sigma'_1 < \sigma_1$. The first part of the proposition then follows noting that $h, H/L, \tau, \widehat{K}$, and r are decreasing functions of σ.

To prove the second part note, first, that at any steady state we have $\widehat{h} = 0$, which implies $\hat{H} = \hat{K} - \hat{A}_L$. For small perturbations around the equilibrium, we can therefore analyze the

is likely to flatten out, so that (provided τ_0 is not too high) a stable equilibrium is attained.

[29]Proposition 2 holds under the assumption that $\sigma_1 > \sigma_{\min}$. If $\sigma_{\min} \geq \sigma_1 > 1$, then A5 implies that the change in θ has no effects on the economy, which is caught in a low-skill trap. Intuitively, if there are no people who wish to get educated because the skill premium is too low, increasing access to education has no effect. The existence of a low-skill trap is consistent with results in the standard growth literature, such as Galor and Tsiddon (1997). (Observe, however, that by Proposition 2, the government can still foster growth and reduce intra-workers inequalities by increasing the fraction of H-workers that it employs, ε. Yet, this has no effect on the growth rate of human capital, which remains stationary.)

change in the steady state value of $\hat{H}$ by focusing on the expression

$$\frac{d\left(\hat{K}-\hat{A}_L\right)}{dh}=s\lambda\sigma(h)\left[(1-\rho)\frac{k^\rho}{k^\rho-(\mu(1-\varepsilon)h)^\rho}-\varepsilon\right]-\tau_1\mu.$$

By A1 as $h\to\frac{k}{\mu(1-\varepsilon)}$, we have $\frac{d\left(\hat{K}-\hat{A}_L\right)}{dh}\to\infty$ and as $h\to\infty$ $\frac{d\left(\hat{K}-\hat{A}_L\right)}{dh}\to-\tau_1\mu$, and provided ε is not too large, $\frac{d^2\left(\hat{K}-\hat{A}_L\right)}{dh^2}<0$. Therefore there is a unique cut-off value h^* such that for all $h<h^*$, $\frac{d\left(\hat{K}-\hat{A}_L\right)}{dh}>0$ and for all $h>h^*$, $\frac{d\left(\hat{K}-\hat{A}_L\right)}{dh}<0$. The desired result then follows noting that at the new steady state the equilibrium change in $\hat{H}$ must be equal to the change in $\hat{K}-\hat{A}_L$.

2. Consider next ε. The desired argument follows in a similar fashion noting that, for any given h, an increase in ε yields an increase in $\sigma(h)$ and $c(\sigma(h))$, which implies in turn an upward shift in the $\hat{h}$ schedule for all h. ■

The dynamics of the economy can be examined graphically. Figure 2 shows the growth rates of the main variables, as functions of h, under condition (KTB). The first two terms on the right hand side of equation (16) show the growth rate of labor productivity, A_L, which increases with h. The third term represents the growth rate of the number of skilled workers, H: as h increases, this rate falls as the skill premium falls. The last term represents the growth rate of the capital stock, K. An increase in h implies that the skill premium falls, so that the rate of profit rises because total labor costs decrease. We add up the rates of growth of A_L and H to obtain the $\hat{A}_L+\hat{H}$ curve, which first decreases and then eventually increases with h. The long-run equilibria are determined at $h_1=b(\sigma_1)(1-\varepsilon)^{-1}$ and $h_2=b(\sigma_2)(1-\varepsilon)^{-1}$, where $k/\mu(1-\varepsilon)<h_1<h_{min}$, and $h_{min}=b(\sigma_{min})(1-\varepsilon)^{-1}$.

The long-run equilibrium h_1 is stable: starting from $h<h_1$, for instance, $\hat{A}_L+\hat{H}>\widehat{K}$, so that h increases till it reaches h_1. The long-run equilibrium h_2, instead, is unstable: if $h>h_2$

initially, h increases indefinitely, with physical and human capital growing very slowly, or not at all, and the growth rate of technical change dominating the dynamics.

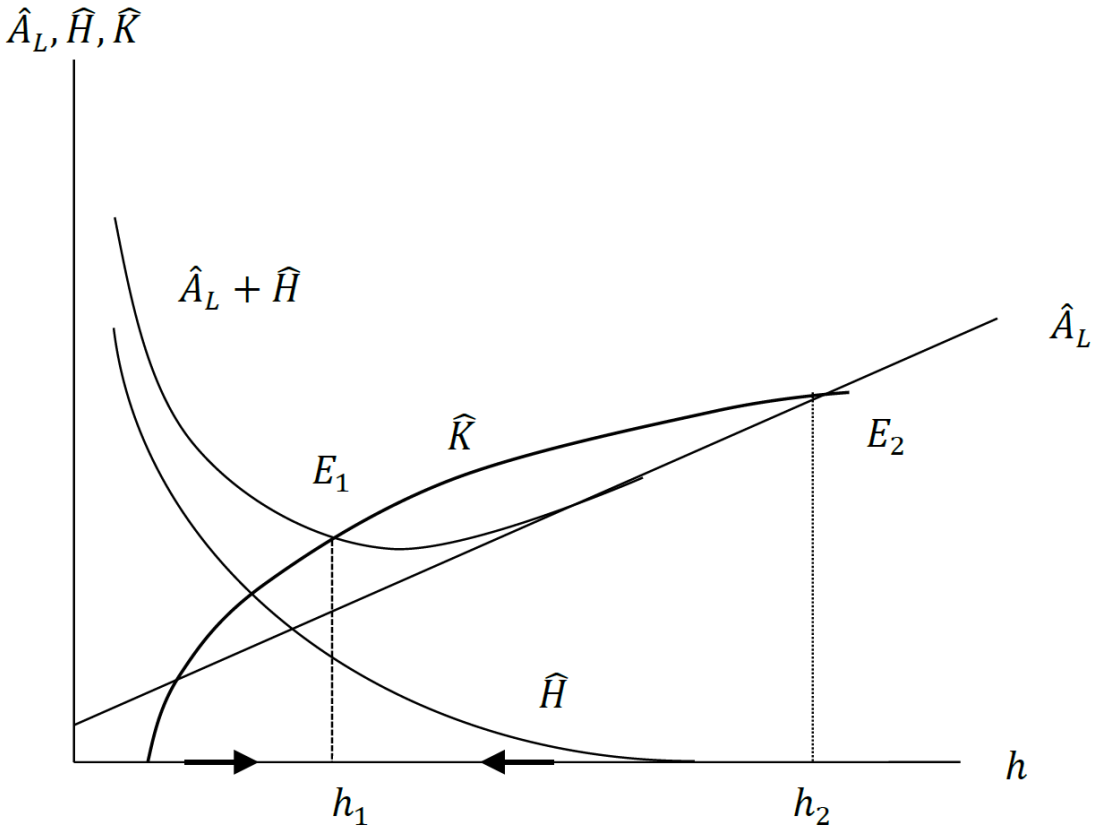

Figure 2: The long-run dynamics and equilibrium.

Focusing on the stable long-run equilibrium E_1, an increase in the openness of the education system – captured by an increase in θ – shifts the $\hat{H}$ and $\hat{A}_L + \hat{H}$ curves upwards, making them steeper, at all values h such that $k/\mu(1-\varepsilon) < h < h_{min}$. The long-run equilibrium growth rates of capital and labor productivity increase, as can be verified from figure 2.

Consider next the effects on distribution. The low-skilled income share, $\lambda c(\sigma)/k$, de-

creases because the number of L-workers employed is a positive function of σ, given the bargaining parameter and the productivity of capital. The high-skilled income share is given by $\sigma(h)\lambda h/k(1-\varepsilon)$. When θ increases, the long-run equilibrium level of h increases, but the rise in h is proportionately less than the fall in σ, given the inelasticity of the $b(.)$ function, so that $\sigma(h)h$ falls. Indeed, the overall wage share decreases and both the rate of profit and the profit share, r/k, rise. Thus income is redistributed from workers to capitalists. This, of course, is why capital accumulation is speeded up, as we saw earlier. However, intra-workers inequalities decline as H-workers lose more than L-workers and their relative share falls.

The effects on workers' welfare are also interesting. The growth rate of the real wage of both types of workers rises with a higher rate of technological change. Further, note that the growth rate of low-skilled employment is $\hat{L} = \hat{c}(\sigma) + \hat{K} - \hat{A}_L$. Since at a steady state $\hat{c}(\sigma) = 0$, and $\hat{H} = \hat{K} - \hat{A}_L$, then $\hat{H} = \hat{L}$, and so the growth rates of the two types of labor coincide in long-run equilibrium. For low levels of h_1 a small increase in θ causes a rise in the growth rate of human capital and employment. So the positive direct effect on the education system dominates the negative indirect effect via the decrease in the long run wage differential. For high levels of h_1, the opposite is true. Thus, for low h_1 the condition of L-workers, in terms of their real wage level and growth rate, and employment growth, is unequivocally improved.[30] H-workers, instead, lose out in the sense that the equilibrium wage differential decreases. But they do gain from the increase in the steady state growth rates of the high-skilled real wage and employment.

Although our main focus is on education, it is interesting to analyze the effect of an increase in the productivity differential between H- and L-labor, μ, which captures the effect

[30] Another potential benefit for the low skilled derives directly from increased access to education leading to a change of status, and of wage, even though this effect is difficult to quantify.

of skill-biased technical change in our model.[31] Such a change decreases the demand for both types of labor at a given h, due to the limited substitutability between them, thus increasing the profit rate (since firms maximize profits), shifting the $\widehat{K}$ curve upwards. The decrease in the demand for H-labor lowers σ at a given h, decreasing $\hat{H}$ and shifting its curve down. However, the increase in μ increases $\hat{A}_L$ for a given h shifting its curve upwards. Although the rate of capital accumulation must increase, overall the effects of skill-biased technical change are, interestingly, unclear. If the effect on $\widehat{K}$ and $\hat{H}$ dominates, then skill-biased technical change leads to an increase in the skill premium and intra-worker inequality, and a decrease in the profit share. However, if technological change responds strongly to h, and the effect on $\hat{A}_L$ dominates, skill-biased technical change may lead to a *reduction* in the skill premium, and an increase in the profit share, contrary to the received wisdom.

Given our focus on growth and distribution, a natural question to ask at this point would concern the effect of a change in the workers' bargaining power. However, the analysis of a purely exogenous perturbation of the class struggle parameter, λ, is not very informative. Given the relevance of education, the existence of two separate groups of workers with potentially conflicting interests, and the endogenous determination of the skill composition of labor, it is important to analyze the interplay of education and labor market conditions.

[31] We note in passing that a sufficiently small improvement in the conditions of technical change, (τ_0, τ_1), has the same qualitative effects as an improvement in the openness of the educational system except that the growth rate of human capital and employment definitely declines. Instead an increase in the capitalists' savings rate, s, raises the skill premium leading to an increase in intra-workers inequality and a decrease in the profit share. Therefore, interestingly, it has an ambiguous effect on the growth rate of the economy and a negative effect on the growth rate of technical change.

5 Education, growth and distribution with variable conditions of class struggle

We have so far assumed a constant low wage-productivity ratio, λ, given by bargaining conditions. This assumption may be questionable, given the strong evidence of short-term fluctuations in income shares driven by distributive conflict (Barbosa-Filho and Taylor 2006; Flaschel 2009). To examine the implications of changes in λ we assume that L-workers have a target ratio between wages and productivity which they try to achieve by pushing up their real wage, but they are not fully able to increase their real wage at the same rate as productivity growth. Formally:

Assumption 8 (A8). The real wage of L-workers changes according to

$$\widehat{w}_L = \delta_1(\lambda^* - \lambda) + \delta_2\tau, \tag{17}$$

where $\delta_1 > 0$, $\lambda^* = \lambda^*(\theta)$, $\delta_2 = \delta_2(\theta)$, $1 > \delta_2 > 0$, and $\delta_2(.), \lambda^*(.)$ are differentiable functions.

The value λ^* is the target ratio of L-workers, which may be interpreted as reflecting normative considerations (e.g. criteria of 'just pay'). The value δ_2 reflects workers' bargaining strength and cohesion, and thus their ability to claw back the benefits of productivity increases.

Equation (17) can be interpreted as the reduced form of a more complex dynamic model with bargaining and optimizing agents (see, for example, Tavani 2013), and its parameters capture broad economic, social, and institutional factors affecting workers' norms, attitudes, and beliefs, and more generally bargaining conditions.[32] Thus, we assume that both the

[32]Including, for example, labor market conditions, which are not modeled explicitly here, consistent with A3 and the existence of an unlimited supply of L-labor.

target ratio λ^* and the parameter capturing worker's bargaining power, δ_2, are affected by the educational system, as captured by the educational access parameter, θ.

This assumption is consistent with the burgeoning literature on the effect of norms, beliefs, and ideology on macroeconomic performance and distributive conflict (e.g., Alesina and Glaeser 2004; Bénabou and Tirole 2006; Galor and Moav 2006). There is ample evidence of violations of the rational expectation hypothesis, with agents holding distorted, or a priori unfounded, perceptions of economic facts and often engaging in costly dissonance-reduction when faced with facts that contradict their views. Agents' views and perceptions influence their choices and actions, and thus macroeconomic outcomes, and may lead to multiple equilibria in which beliefs and macroeconomic outcomes are self-sustaining. For example, empirical evidence shows that "individual voters' perceptions of the extent to which people control their own fate are major determinants of their attitudes toward inequality and redistribution" (Bénabou and Tirole 2006, p.702). The educational system clearly plays a central role here in that it contributes to shape norms, attitudes, and beliefs.

Alesina and Glaeser (2004), for example, argue that the differences between what may be called the American and the European varieties of capitalism are at least partly explained by differences in norms and beliefs. They also argue that "American beliefs result from indoctrination predominantly controlled by the wealthier classes, whereas European beliefs result from indoctrination predominantly controlled by Marxist-influenced unions, teachers, and politicians" (Bénabou and Tirole 2006, p.703). In their historical analysis of the role of education and human capital in the development of capitalist economies, Galor and Moav (2006) establish an even more direct connection between educational systems and the behavior of the working class. They argue that in all capitalist economies the establishment of a public education system was originally "motivated by a variety of reasons such as religion,

social control, moral conformity, socio-political stability, and military efficiency" (Galor and Moav 2006, p.89), and was meant to inculcate values of punctuality, discipline, manners, and diligence in the working classes.

A8 can thus be seen as a succinct way of capturing the relevance of education, and educational systems – surely, together with other social, cultural, and institutional factors, – in shaping individual norms, attitudes, and beliefs, which in turn affect behavior in distributive conflicts and, ultimately, distributive outcomes.

It may be objected that equation (17) concerns the dynamics of the *low-skilled* workers' wage in relation to a measure of *low-skilled* labor productivity, but it is not clear why the structure of the higher education system should affect the bargaining strength of workers who do not actually access it. We believe that the effect permeates into the entire working class, including those who do not actually make use of higher education. In particular, the openness of higher education can be related to changes in the quality of basic education which affects all workers, making L-workers academically more prepared thus increasing their chances of admission to, and success in, higher education.

Perhaps more importantly, the bargaining power of L-workers arguably depends on the cohesion of the working class as a whole and therefore also on the attitude of H-workers. And H-workers have *two* possible avenues to improve their economic conditions, for their salary depends *both* on the skill premium *and* on the L-workers' wage which represents the base wage of H-workers. The 'individualistic' avenue focuses on the skill premium which is determined competitively in the labor market. However, they can also adopt a 'collective', class-based strategy to improve their lot by supporting L-workers in the bargaining process. The two avenues are not mutually exclusive and their salience for H-workers will depend on a number of institutional, normative and even cultural factors, which are also affected by

the education system.

In other words, education affects workers' solidarity and the ability of workers of both types to maintain their wages in line with productivity increases; and there are "externalities" from the education of H-workers to L-workers.

While we assume that educational access influences λ^* and δ_2, we do not specify exactly how it affects workers' attitudes in bargaining and conflict in order to allow for fundamental differences in the nature of educational systems, consistent with the literature discussed earlier. Below, we consider two alternative scenarios.

A8 states that the growth rate of the real wage depends positively on the extent to which the actual ratio λ falls short of the target, and on the growth of labor productivity, because workers demand and receive at least part of the fruits of higher labor productivity.[33] Since, from the definition of λ we have

$$\widehat{\lambda} = \widehat{w}_L - \tau, \tag{18}$$

substituting equations (5a) and (17) into (18) we get

$$\widehat{\lambda} = \delta_1(\lambda^* - \lambda) - (1 - \delta_2)(\tau_0 + \tau_1 \mu h). \tag{19}$$

Thus, an increase in h, by increasing the rate of productivity growth, reduces the growth rate of the wage-productivity ratio because workers are unable to increase their real wage to capture the full gains from productivity growth. An increase in the wage-productivity ratio reduces its rate of increase because workers are closer to their target.

Equations (16) and (19) form a dynamic system involving the two state variables h and

[33]These dynamics could be made to depend on inflationary dynamics of price and money wage changes, but we abstract from these complications for simplicity. See, for instance, Dutt (1990) for a discussion.

λ. Equation (19) implies that the $\widehat{\lambda} = 0$ isocline is:

$$\lambda = \lambda^* - \frac{1-\delta_2}{\delta_1}(\tau_0 + \tau_1 \mu h). \tag{20}$$

Consider the $\widehat{h} = 0$ isocline. Consistent with the analysis in the previous section, we assume that there exists *some* positive value of λ such that (KTB) holds. Formally:

$$\tau_0 + \tau_1 k \left[M+1\right]^{-\frac{1}{\rho}} (1-\varepsilon)^{-1} < sk. \ \ (KTB_0)$$

(KTB_0) ensures the existence of a stationary value of h with positive real wages and a skill premium greater than one if $\lambda = 0$. Then there exists a value of λ, denoted as $\overline{\lambda}$, that solves $\tau_0 + \frac{\tau_1 k}{1-\varepsilon}\left[M+1\right]^{-\frac{1}{\rho}} = sk\left[1 - \bar{\lambda}\mu^{-1}\left[M+1\right]^{-\frac{1}{\rho}}\left[M + \frac{1}{1-\varepsilon}\right]\right]$. By Proposition 1, we know that for all $\lambda \in [0, \overline{\lambda})$, there exist two values (h_1, h_2), with $h_1 < \overline{h} = b(1)(1-\varepsilon)^{-1} = k\mu^{-1}\left[M+1\right]^{-\frac{1}{\rho}}(1-\varepsilon)^{-1} < h_2$, such that $\widehat{h} = 0$, whereas if $\lambda = \overline{\lambda}$, then $h = \overline{h} = b(1)(1-\varepsilon)^{-1}$ implies $\widehat{h} = 0$. We also know that, for all $\lambda \in [0, \overline{\lambda})$, at h_1, $\frac{d\hat{h}}{dh} < 0$ whereas at h_2, $\frac{d\hat{h}}{dh} > 0$ and the $\widehat{h} = 0$ isocline is first increasing and then decreasing in h.[34] From equation (16) we see that an increase in λ reduces the rate of profit and the rate of accumulation by increasing the payments to both kinds of workers, and hence increases $\widehat{h}$. At h_1, the effect of h on physical and human capital accumulation is stronger than the effect on technological change, and an increase in h reduces $\widehat{h}$, so that the $\widehat{h} = 0$ isocline is positively sloped. At h_2, the opposite holds, and the $\widehat{h} = 0$ isocline is negatively sloped.

Let $\lambda^{max} = \lambda^* - \frac{1-\delta_2}{\delta_1}\tau_0$ and $h^{max} = \frac{\delta_1}{\tau_1\mu(1-\delta_2)}\lambda^* - \frac{\tau_0}{\tau_1\mu}$ denote the intercepts of the $\widehat{\lambda} = 0$

[34]Indeed, the argument in Proposition 1 can be generalized to prove the existence of $\widetilde{\lambda} \geq \overline{\lambda}$ such that for all $\lambda \in [0, \widetilde{\lambda})$, there exist two equilibria (h_1, h_2) such that $h_1 < h_2$ and at h_1 $\frac{d\hat{h}}{dh} < 0$ whereas at h_2, $\frac{d\hat{h}}{dh} > 0$. It can also be proved that the downward sloping part of the $\widehat{h} = 0$ isocline is concave in λ.

isocline. Let $h_1(0)$ and $h_2(0)$ denote the two values of h, with $0 < h_1(0) < h_2(0)$, such that $\widehat{h} = 0$ when $\lambda = 0$: by Proposition 1 they exist, and $h_1(0) = b(\sigma_1)(1-\varepsilon)^{-1}$ with $\sigma_1 > 1$ and $h_2(0) = b(\sigma_2)(1-\varepsilon)^{-1}$, with $\sigma_2 < 1$. By A5, $h_2(0) = (sk - \tau_0)/\tau_1\mu$.

Proposition 3 provides sufficient conditions for the existence of economically meaningful long-run equilibria.

Proposition 3 ***(Long-run equilibria)***. *Assume A1-A8. Suppose that (KTB_0) holds and* $0 < \lambda^{max} < \overline{\lambda}$.

(i) If $h^{max} \geq h_2(0)$*, then there exist two long-run equilibria:* E_1 *is stable with* $\sigma_1 > 1$ *and* E_2 *is unstable with* $\sigma_2 < 1$*. At* E_1 *the wage-productivity ratio is higher, and the stock of human capital is lower than at* E_2.

(ii) If $h_1(0) \leq h^{max} < h_2(0)$*, there exists a unique stable long-run equilibrium with* $\sigma_1 > 1$.

Proof. 1. Claim (i). Suppose that $0 < \lambda^{max} < \overline{\lambda}$ and $h^{max} \geq h_2(0)$. The existence of the two equilibria follows noting that given the monotonicity of the two isoclines and the concavity of the downward sloping part of the $\widehat{h} = 0$ isocline, the two curves intersect twice: once in the increasing part of the $\widehat{h} = 0$ isocline, at E_1, and once in its decreasing part, at E_2. By Proposition 1, and the shape of the two isoclines, it follows that at E_1 $\sigma_1 > 1$, while at E_2 $\sigma_2 < 1$, and so $h_1 < h_2$. Furthermore, since the $\widehat{\lambda} = 0$ isocline is negatively sloped it follows that $\lambda_1 > \lambda_2$.

2. In order to investigate the stability properties of the two equilibria consider the Jacobian of the dynamic system

$$\begin{bmatrix} \frac{d\widehat{h}}{dh} = \tau_1\mu + \varepsilon\theta g'(\sigma)\sigma'(h) + s\lambda\sigma(h)\left[\varepsilon - (1-\rho)\frac{m(h)}{m(h)-1}\right] & \frac{d\widehat{h}}{d\lambda} = s\left[c\left(\sigma(h)\right) + \sigma(h)h\right] > 0 \\ \frac{d\widehat{\lambda}}{dh} = -(1-\delta_2)\tau_1\mu < 0 & \frac{d\widehat{\lambda}}{d\lambda} = -\delta_1 < 0 \end{bmatrix}$$

where $m(h) = \left(\frac{k}{\mu(1-\varepsilon)h}\right)^{\rho}$. Since the $\widehat{h} = 0$ isocline is positively sloped at E_1 and negatively sloped at E_2, the upper left entry is negative in the former case and positive in the lat-

ter. Therefore in the neighborhood of E_1 the Jacobian has a negative trace and a positive determinant, satisfying the conditions for stability. In a neighborhood of E_2, instead, the determinant of the Jacobian is negative, so that the equilibrium is a saddle-point.

3. If $h_1(0) \leq h^{max} < h_2(0)$, then given the monotonicity of the two isoclines, they intersect only once on the upward sloping part of the $\widehat{h} = 0$ isocline, E_1. As in step 1, it follows that $\sigma_1 > 1$. The stability of the long-run equilibrium E_1 follows as in step 2. ■

Remark: Under (KTB_0), there always exist combinations of the parameters such that the conditions $0 < \lambda^{max} < \overline{\lambda}$ and either $h_1(0) \leq h^{max} < h_2(0)$ or $h^{max} \geq h_2(0)$ can both hold.[35]

Figure 3 shows the long-run dynamics of this model.[36] The long-run equilibria occur at the intersections of the $\widehat{h} = 0$ and $\widehat{\lambda} = 0$ curves. The economically meaningful equilibrium E_1 is (asymptotically) stable, and the economy may converge directly or cyclically to it, as can be seen from the arrows. The equilibrium E_2 instead is unstable. The possibly cyclical movement around E_1 is quite interesting and consistent with classical models of growth with distributive conflict such as Goodwin's (1967) seminal contribution.

We may now analyze the effect of education in the economy. Consider first the $\widehat{h} = 0$ isocline. Proposition 2 implies that an increase in θ shifts (a portion of) the upward sloping part of the $\widehat{h} = 0$ curve to the right, as shown by the dotted line, because it increases $\widehat{h}$ at given values of h and λ, for all h such that $\sigma = \sigma(h) > \sigma_{min}$.

The effect of an increase in θ on the $\widehat{\lambda} = 0$ isocline depends on the nature of education and on its effect on the workers' bargaining position. If education facilitates the self-development

[35]Observe, however, that both λ^{max} and h^{max} are monotonically increasing in λ^*, δ_1, and δ_2. Therefore, if conditions in the labor market are particularly conflictive, it may happen that the $\widehat{\lambda} = 0$ isocline lies entirely above the $\widehat{h} = 0$ isocline and no equilibrium exists.

[36]Figure 3 has been drawn assuming the $\widehat{h} = 0$ isocline to be strictly concave. This is for the purposes of illustration and none of the results depend on it.

of individuals, making them more conscious of their rights and nature as social beings, and increases workers' solidarity, then the functions $\lambda^*(\theta)$ and $\delta_2(\theta)$ may be increasing in θ. In this case, the $\widehat{\lambda} = 0$ isocline tends to shift to the right and become flatter. The combined effect on the two curves can be summarized in the next proposition.

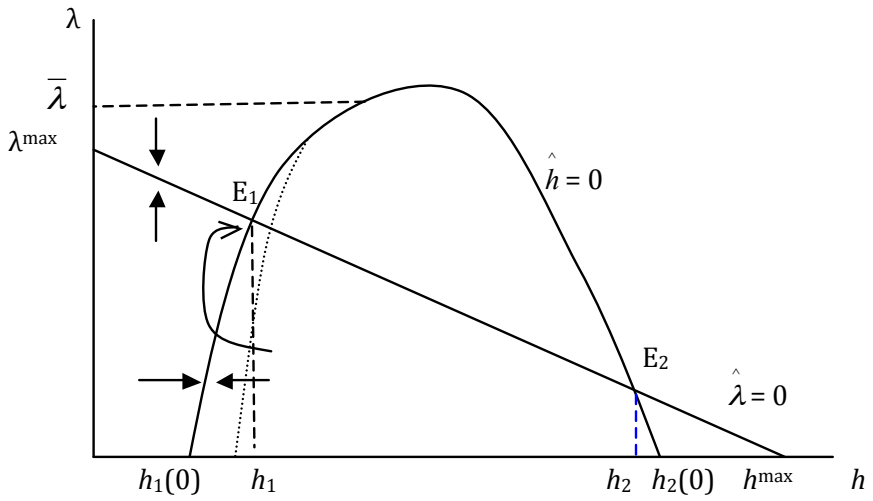

Figure 3: Long-run dynamics and equilibria of the model with variable λ.

Proposition 4 ***(Progressive role of education)***. *Assume A1-A8. Suppose that* (KTB_0) *holds. Suppose that* $0 < \lambda^{max} < \overline{\lambda}$, $h^{max} \geq h_1(0)$, *and* $\lambda^*(\theta), \delta_2(\theta)$ *are increasing in* θ. *Then at a long-run equilibrium with* $\sigma_1 > \sigma_{min}$, *a small increase in* θ *yields a decrease in intra-worker inequality, whereas the skill composition of the labor force and the growth rate of technological progress increase. Further, if education has a strong effect on workers' values, beliefs and expectations, their bargaining strength increases.*

Proof. 1. By Proposition 3, there exists at least one long-run equilibrium with $\sigma_1 > 1$.

Suppose that $\sigma_1 > \sigma_{min}$. Consider the two isoclines

$$\tau_0 + \tau_1\mu h + \theta\varepsilon g(\sigma(h)) - s\left[k - \lambda c(\sigma(h)) - \sigma(h)\lambda h\right] = 0 \quad (21)$$

$$\delta_1(\lambda^* - \lambda) - (1-\delta_2)(\tau_0 + \tau_1\mu h) = 0 \quad (22)$$

Let $W(h) = c(\sigma(h)) + \sigma(h)h$. By totally differentiating the latter expressions with respect to h, λ and θ, we obtain

$$\begin{aligned} \frac{d\widehat{h}}{dh}dh + sW(h)d\lambda + \varepsilon g(\sigma(h))d\theta &= 0 \\ \frac{(1-\delta_2)}{\delta_1}\tau_1\mu dh + d\lambda - \left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]d\theta &= 0 \end{aligned}$$

where $\frac{d\widehat{h}}{dh} = \tau_1\mu + \varepsilon\theta g'(\sigma)\sigma'(h) + s\lambda\sigma(h)\left[\varepsilon - (1-\rho)\frac{m(h)}{m(h)-1}\right]$ and $m(h) = \left(\frac{k}{\mu(1-\varepsilon)h}\right)^{\rho}$. Writing the latter system of equations in matrix form:

$$\begin{bmatrix} \frac{d\widehat{h}}{dh} & sW(h) \\ \frac{(1-\delta_2)}{\delta_1}\tau_1\mu & 1 \end{bmatrix} \begin{bmatrix} dh \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\varepsilon g(\sigma(h)) \\ \left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right] \end{bmatrix} d\theta. \quad (23)$$

Let $\Delta = \frac{d\widehat{h}}{dh} - sW(h)\frac{(1-\delta_2)}{\delta_1}\tau_1\mu$, equation (23) can also be written as

$$\begin{bmatrix} dh \\ d\lambda \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} 1 & -sW(h) \\ -\frac{(1-\delta_2)}{\delta_1}\tau_1\mu & \frac{d\widehat{h}}{dh} \end{bmatrix} \begin{bmatrix} -\varepsilon g(\sigma(h)) \\ \left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right] \end{bmatrix} d\theta.$$

Therefore the effect of an increase in the openness of the education system on the two

state variables can be expressed as follows:

$$\frac{dh}{d\theta} = -\frac{1}{\Delta}\left\{\varepsilon g(\sigma(h)) + sW(h)\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]\right\} \tag{24}$$

$$\frac{d\lambda}{d\theta} = \frac{1}{\Delta}\left\{\varepsilon g(\sigma(h))\frac{(1-\delta_2)}{\delta_1}\tau_1\mu + \frac{d\widehat{h}}{dh}\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]\right\}. \tag{25}$$

At the long run equilibrium with $\sigma_1 > 1$, we know from Proposition 3 that $\frac{d\widehat{h}}{dh} < 0$, and therefore noting that $W(h) > 0$, it follows that $\Delta < 0$.

2. If education is progressive, then $\frac{d\lambda^*}{d\theta} > 0$, $\frac{d\delta_2}{d\theta} > 0$ and $\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right] > 0$. Therefore $\frac{dh}{d\theta} > 0$. The effect on λ cannot be definitely signed but $\frac{d\lambda}{d\theta} > 0$ if and only if the following condition holds:

$$\varepsilon g(\sigma(h))\frac{(1-\delta_2)}{\delta_1}\tau_1\mu < -\frac{d\widehat{h}}{dh}\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]. \tag{26}$$

Therefore, the lower ε, $g(\sigma(h))$, τ_1, μ , the higher δ_1, δ_2, and especially $\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]$, the more likely the latter inequality will hold. ■

If, instead, education is important in inculcating an ideology of resignation and moderation, in increasing the tolerance for inequality and creating the perception of greater upward mobility than actually exists, or in undermining workers' solidarity by emphasizing differences in educational attainment, then the functions $\lambda^*(\theta)$ and $\delta_2(\theta)$ may be decreasing in θ. In this case, the $\widehat{\lambda} = 0$ isocline will tend to shift to the left and to become steeper. Then:

Proposition 5 ***(Education as ideology)***. *Assume A1-A8. Suppose that (KTB$_0$) holds. Suppose that $0 < \lambda^{max} < \overline{\lambda}$, $h^{max} > h_1(0)$, and $\lambda^*(\theta)$ and $\delta_2(\theta)$ are decreasing in θ. Then at a long-run equilibrium with $\sigma_1 > \sigma_{min}$, a small increase in θ yields a decrease in workers' bargaining strength. Further, if education has a strong effect on workers' values, beliefs and*

expectations, the wage differential increases.

Proof. Equations (24)-(25) can be derived as in the proof of Proposition 4. Then, if education is regressive, then $\frac{d\lambda^*}{d\theta} < 0, \frac{d\delta_2}{d\theta} < 0$ and $\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right] < 0$. Therefore $\frac{d\lambda}{d\theta} < 0$. The effect on h cannot be definitely signed but $\frac{dh}{d\theta} < 0$ if and only if the following condition holds:

$$\varepsilon g(\sigma(h)) < -sW(h)\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]$$

or, equivalently,

$$\varepsilon g(\sigma(h)) < -s\left[c(\sigma(h)) + \sigma(h)h\right]\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]. \tag{27}$$

Therefore, the lower ε and $g(\sigma(h))$, the higher s, labor costs, and especially $\left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]$, the more likely the latter inequality will hold. ■

Thus, if education has a significant effect on workers' attitudes and beliefs, more proactive education policies have an impact on income distribution *within the working class*. If education is progressive, workers' bargaining power increases, wage differentials decrease (as an effect of increased workers' solidarity) and the steady state growth rates of H- and L-workers' real wages increase, thanks to the higher growth rate of technological progress. The opposite holds if education deepens any cleavages within the working class.

Interestingly, however, the effect on the distribution of income *between wages and profits* – and thus on growth – is not obvious, because the bargaining strength of the working class as a whole and wage differences within the working class may move in opposite directions.

Proposition 6 *Assume A1-A8. Suppose that (KTB$_0$) holds. Suppose that* $0 < \lambda^{max} < \overline{\lambda}$

and $h^{max} > h_1(0)$. *Let* $W(h) = c(\sigma(h)) + \sigma(h)h$ *and* $\frac{dB}{d\theta} = \left[\frac{d\lambda^*}{d\theta} + \frac{1}{\delta_1}(\tau_0 + \tau_1\mu h)\frac{d\delta_2}{d\theta}\right]$. *At a long-run equilibrium with* $\sigma_1 > \sigma_{min}$, *a small increase in* θ *yields an increase in the profit rate, in the profit share, and in the growth rate of the economy if and only if the following condition holds:*

$$\varepsilon g(\sigma(h)) \left[\frac{(1-\delta_2)}{\delta_1}\tau_1\mu W(h) - \lambda W'(h)\right] + W(h)\left[\tau_1\mu + \varepsilon\theta g'(\sigma(h))\sigma'(h)\right]\frac{dB}{d\theta} > 0. \qquad (28)$$

Proof. By Proposition 3, there is at least one long-run equilibrium with $\sigma_1 > 1$. Suppose that $\sigma_1 > \sigma_{min}$. Equations (24)-(25) can be derived as in the proof of Proposition 4.

For given s and k, the profit rate, the profit share and the growth rate of the economy increase in equilibrium if and only if total labor costs $\lambda W(h)$ decrease:

$$\frac{d\lambda}{d\theta}W(h) + \lambda W'(h)\frac{dh}{d\theta} < 0. \qquad (29)$$

Using equations (24) and (25), the latter inequality becomes

$$\frac{1}{\Delta}\left\{\varepsilon g(\sigma(h))\frac{(1-\delta_2)}{\delta_1}\tau_1\mu + \frac{d\widehat{h}}{dh}\frac{dB}{d\theta}\right\}W(h) - \lambda W'(h)\frac{1}{\Delta}\left\{\varepsilon g(\sigma(h)) + sW(h)\frac{dB}{d\theta}\right\} < 0.$$

Equation (28) follows rearranging the latter expression and noting that $\frac{d\widehat{h}}{dh} = \tau_1\mu + \varepsilon\theta g'(\sigma)\sigma'(h) + s\lambda W'(h)$ and $\Delta < 0$. ■

By Proposition 6, it follows that if education has limited effects on workers' attitudes, norms and beliefs (conditions (26) and (27) do *not* hold), then an increase in the openness of the education system fosters profitability and growth. If, instead, education has a strong effect on workers' values, beliefs and expectations the effect of an increase in education is less clear cut. For example, if education is progressive, the skill differential and intra-

workers inequalities decrease, which tends to reduce the share of wages in national income. If education has a strong effect in influencing workers' attitudes, this may be compensated by an increase in workers' bargaining power, which increases the wages of *both* types of workers. The overall effect on the profit rate and the profit share depends on the relative strength of the two effects as captured in condition (28).[37]

6 Back to the stylized facts

The previous analysis shows that a classical approach produces some interesting and novel insights on the economic role of education. It may be objected, however, that the full model does not allow us to derive definite conclusions on the effects of education on growth and distribution, which depend on the characteristics of the educational system and on other social and institutional factors. We see this apparent indeterminacy as an advantage of our model. Theoretically, it is inappropriate to analyze the effects of "education" as if it was an objectively defined, neutral notion and abstracting completely from its interaction with the broader structure of the labor market and employment relations. Empirically, noting the major international differences in educational systems, labor market institutions, and so on, our approach allows one to formulate testable claims that are appropriate for given

[37]The effect on the growth rates of human capital and employment is also ambiguous. The effect of an increase in education on the growth rate of human capital is given by $\frac{d\widehat{H}}{d\theta} = \varepsilon g(\sigma(h)) + \varepsilon\theta g'(\sigma(h))\sigma'(h)\frac{dh}{d\theta}$. Therefore, using equation (24), an increase in θ increases the growth rate of human capital – and employment – if and only if $g(\sigma(h)) > \frac{\theta g'(\sigma(h))\sigma'(h)}{\Delta}\left[\varepsilon g(\sigma(h)) + sW(h)\frac{dB}{d\theta}\right]$. This inequality always holds if education is regressive and it has a strong effect on workers' norms and beliefs (so that condition (27) holds and the expression in square brackets is negative). If, however, condition (27) does not hold, or education is progressive, then a more open education system speeds up the accumulation of human capital only if the direct effect of the increase in θ more than compensates the indirect effect via a decrease in the skill premium.

countries. Our analysis, for example, highlights an interesting causal mechanism that may contribute to explain some of the stylized facts of the US economy since 1975.

To use our analysis to understand the trends in growth and income distribution we note that while in *quantitative* terms education has expanded, the *nature* of education in the US (and other countries) has changed over the past few decades. Although changes in the nature of education are difficult to demonstrate with hard data, qualitative judgements made by observers of education systems from a variety of perspectives, as well as some broad quantitative indicators, are revealing and suggest that education has taken an increasingly regressive character.

The class-based nature of schooling in the US has long been highlighted by radical and Marxian scholars, who have criticized both the type of education provided and its unequalizing effect due to the differential treatment of students from different classes (in addition to the different use students can make of educational opportunities because of family circumstances, something that has been discussed more widely). In their seminal work, for example, Bowles and Gintis (1975, 1976) have shown that schools, colleges and universities socialize "students to accept beliefs, values and forms of behavior on the basis of authority, rather than the students' own critical judgments of their interests" (Bowles and Gintis 2002, p.12). They have also examined how these qualities can be traced to a variety of factors, including the role of the capitalist class in influencing the formation and growth of educational institutions and their curricula. Baran and Sweezy (1966, pp.333-4) have discussed how high schools in the US differentiate between the "academically talented" and others. They cite the view of Harvard University President James Conant who, after a comprehensive inquiry into the educational system felt that the former, who comprised between 15 to 20 percent of all high-school students, "should be given a break: they should be more challenged, their

program of academic subjects should be intensified and broadened." To the remaining 80 to 85 per cent, Conant's approach is quite different. His prescription for the plebs is "meaningful sequences of courses leading to the development of marketable skills ... and while the students enrolled should also get some instruction in English, social studies and the like, no undue stress is to be placed in their curricula on academic subjects".

From a liberal perspective, Nussbaum (2010), drawing on the experiences of the US and other countries, points out that "[r]adical changes are occurring in what democracies teach the young ... Thirsty for national profit, nations, and their systems of education, are heedlessly discarding skills that are needed to keep democracies alive. If this trend continues, nations all over the world will soon be producing generations of useful machines, rather than complete citizens who can think for themselves, criticize tradition, and understand the significance of another person's sufferings and achievements" (Nussbaum 2010, p.2). She continues that "[t]he student's freedom of mind is dangerous if what is wanted is a group of technically trained obedient workers to carry out the plans of elites who are aiming at foreign investment and technological development" (Nussbaum 2010, p.21).

A shift in student interest towards career goals and financial rewards is discernable from the available data, although the extent to which the education system contributes to this or results from it is difficult to determine. According to the US Department of Education's Higher Education General Information Survey, the share of undergraduate business majors in total undergraduate degrees awarded increased from 13.7 per cent in 1970-71 to 22.8 per cent in 1990-91 and has remained more or less in the low 20s since then. The American Freshman Survey, which has collected data from 1967, shows that for entering first year students in the US, when asked about "objectives considered to be essential or very important" the percent who checked "developing a meaningful philosophy of life" fell from 87 in 1967 to 45, and

those who checked "being very well off financially" increased from 40 to 82. It is arguable that these trends in the nature of the education system can explain – at least in part – the incorrect perceptions about the extent of inequality especially in the US. For instance, Norton and Ariely (2011) show that US survey respondents believed that agents in the top quintile of the wealth distribution in the US owned 59 per cent of all the wealth and those in the bottom 40 per cent owned 10 per cent, while the top 20 per cent actually owned 84 per cent and the bottom 40 owned 0.3 per cent.

All this may contribute to explain why the remarkable expansion in education occurred in the last four decades has not produced the expected growth-inducing and inequality-reducing effects suggested by our basic model, and predicted by neoclassical macro models. Our hypothesis is that after the collapse of the Keynesian compromise, there has been a breakdown in collective organization and worker solidarity (forcefully shown in the decrease in unionization rates) which has been caused by a number of economic, institutional and cultural factors.[38] The educational system has played an important role in this trend by taking up an increasingly regressive and ideological character. The breakdown in workers' solidarity has led to an increase in intra-workers inequalities, which have more than offset the decrease in skill differential due to the increase in supply of H-workers.[39] The combined effect

[38]Of course, technological change may also have played an important role in the weakening of workers' bargaining position, as suggested, for example, by Acemoglu et al (2001).

[39]It may be argued that while much of the wage inequality in the US in the last four decades is attributable to the upper-tail (e.g. 90/50 ratio) inequality, the labor market institutions analyzed in the paper are capable of explaining the minimum-wage struggles by unions which can only account for lower-tail inequality. This is far from obvious and recent empirical work suggests that labor market institutions have a significant impact on the whole distribution. Jaumotte and Osorio Buitron (2015), for example, "find evidence that the erosion of labor market institutions is associated with the rise of income inequality in our sample of advanced economies, notably at the top of the income distribution."

of the decrease in workers' overall bargaining power and the increase in skill differential has been a relatively stable profit share over the period, which in turn explains (in a profit-led economy such as the US) the relatively flat trend in the economy's growth rate.

It is important to stress that our model is not meant to provide a comprehensive explanation of recent empirical trends, and we are *not* suggesting that the expansion in higher education has caused the increase in inequality and the relatively sluggish growth in the US in the last four decades. The main purpose of our analysis is to provide a counterpoint to the standard emphasis on the beneficial effects of education. For granting that education has taken up an increasingly regressive and ideological character, Proposition 5 does suggest that an increase in education may have contributed – surely, together with many other political, institutional and even cultural factors – to a decrease in workers' bargaining power with the breakdown of workers' solidarity. Education creates skills, but it does many other things too, and education policies should be evaluated taking into account *both* the actual content of the education provided *and* the broader political and institutional framework.

7 Conclusion

This paper has developed a classical model to examine the growth and distributional consequences of education. In the model, an expansion of education allows more L-workers to become H-workers and, in terms of broader political economy considerations, it can affect bargaining conditions. From a theoretical viewpoint, this paper has thus attempted to fill a lacuna in the literature on the classical approach, which has neglected the formal analysis of the effects of education and skill formation on distribution and growth, an issue which seems a key feature of contemporary knowledge-based capitalist economies.

The model suggests that in a profit led, class-divided, economy with significant unemployment, an increase in education may have a positive effect on growth (and on intra-workers inequalities). Yet, unlike in standard growth models this is not due to an improvement in labor productivity but to a worsening distribution between workers and capitalists. Education policies may lead to an increase in the profit share, and in the growth rate, either because an increase in the stock of educated workers lowers the reward to skills (provided that substitutability between two types of workers is not too high), or because education policies can affect workers' norms and beliefs, and thus indirectly their bargaining position.

Our model analyzes how the expansion of education affects the distribution of income between three classes, that is, capitalists, H-workers and L-workers. The expansion of education has the short-run effect of reducing the skill premium and this increases the profit share, given the workers' bargaining power. When we allow the state of class struggle to change endogenously, the distributional effects of an expansion of education depend on the nature of the education system. A progressive education tends to increase workers' bargaining power and solidarity, and to reduce *intra*-class inequalities. The opposite holds if education plays an ideological role of socializing people into accepting large inequalities. Interestingly, however, the effect of education on *inter*-class inequalities cannot be determined a priori: the movement of the profit share depends on the relative strength of the various effects.

In other words, contrary to the received wisdom, in profit-led economies characterized by labor unemployment and wage bargaining, education and human capital accumulation do not yield unambiguously positive effects on distribution. In the presence of several departures from the abstract model with perfectly competitive markets, optimizing agents, and smooth substitutability between inputs of production, more complex and encompassing measures (e.g., improving education access *and* undertaking proactive policies to increase employment)

are necessary to obtain equitable growth outcomes.

The model developed here is simple and various assumptions can be relaxed. Concerning the structure of the economy, it would be interesting to drop the assumption that savings are automatically invested, thus allowing aggregate demand issues to enter into the distribution of output and growth. Further, it would be worth exploring explicitly the implications of demographic factors and of labor supply decisions. Among other things, this would allow us to analyze more explicitly the effect of labor market conditions on workers' bargaining strength in the general model.

Concerning the role of education and its influence on class cleavages, several extensions seem particularly interesting: allowing H-workers to save and hold capital, and thereby have mixed class interests (this case is examined in Dutt and Veneziani (in press) in a special case of the economy considered here); allowing the wage premium to change slowly with the possibility that some H-workers find low-skilled jobs (being chosen above L-workers); and introducing different levels of education (such as primary, secondary, and higher education).[40] These extensions might provide some insights on broader questions regarding the role of education in distributive struggles and in determining the income and wealth distribution. Will the spread of education blur the distinction between workers and capitalists by allowing workers to become capitalists (with human capital)? Or will the classical distinction between workers and capitalists continue to have a central role to play? We leave these issues for further research.

[40]See, for example, Nakajima and Nakamura (2012).

A Addendum (not for publication, to be posted on-line)

A.1 The derivation of the labor demand functions

In this section, for notational simplicity, we shall use H instead of H_P to denote the amount of skilled labor used in productive activities. Given A1, profit maximization yields

$$Y = kK$$

and

$$Y = [(A_L L)^\rho + (A_H H)^\rho]^{1/\rho},$$

where L, H are chosen so as to solve the following problem

$$\begin{aligned} & \min_{L,H} w_L L + w_H H \\ Y^\rho &= (A_L L)^\rho + (A_H H)^\rho. \end{aligned}$$

The Lagrangean is:

$$\Lambda(L, H, \lambda) = w_L L + w_H H + \lambda[Y^\rho - (A_L L)^\rho + (A_H H)^\rho].$$

The first order conditions are:

$$w_H = \lambda\rho A_H^{\rho} H^{\rho-1}, \tag{30}$$

$$w_L = \lambda\rho A_L^{\rho} L^{\rho-1}, \tag{31}$$

$$Y^{\rho} = (A_L L)^{\rho} + (A_H H)^{\rho}. \tag{32}$$

From equations (30) and (31), it follows that

$$\frac{H}{L} = \left(\frac{w_H}{w_L}\right)^{\frac{1}{\rho-1}} \left(\frac{A_H}{A_L}\right)^{-\frac{\rho}{\rho-1}}. \tag{33}$$

Then, by A2, we have $A_H = \mu A_L$, and equation (33) becomes

$$\frac{H}{L} = \left(\frac{w_H}{w_L}\right)^{\frac{1}{\rho-1}} (\mu)^{-\frac{\rho}{\rho-1}} \tag{34}$$

Then, equation (32) can be written as

$$Y^{\rho} = L^{\rho}\left[(A_L)^{\rho} + (A_H H/L)^{\rho}\right] = L^{\rho}\left[(A_L)^{\rho} + (A_H)^{\rho}\left(\frac{w_H}{w_L}\right)^{\frac{\rho}{\rho-1}} (\mu)^{-\frac{\rho^2}{\rho-1}}\right]$$

or, given A2, and using $Y = kK$

$$L = \frac{kK}{A_L\left[1 + \left(\frac{w_H}{w_L}\right)^{\frac{\rho}{\rho-1}} (\mu)^{-\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}}$$

Similarly, using equation (34),

$$H = \frac{kK}{A_L\left[1 + \left(\frac{w_H}{w_L}\right)^{\frac{\rho}{\rho-1}} (\mu)^{-\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} \left(\frac{w_H}{w_L}\right)^{\frac{1}{\rho-1}} (\mu)^{-\frac{\rho}{\rho-1}}$$

or

$$H = \frac{kK}{A_L \mu \left[1 + \left(\frac{w_H}{w_L}\right)^{-\frac{\rho}{\rho-1}} (\mu)^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}}$$

Equations (3) and (4) follow by substituting $\sigma = (w_H/w_L)$.

References

[1] Alonso-Carrera, Jaime, Jordi Caballè and Xavier Raurich (in press) Intergenerational mobility in education and occupation. *Macroeconomic Dynamics*.

[2] Acemoglu, Daron (2002a) Directed Technical Change. *Review of Economic Studies* 69, 781-809.

[3] Acemoglu, Daron (2002b) Technical Change, Inequality and the Labor Market, *Journal of Economic Literature* 40, 7-72.

[4] Acemoglu, Daron, Philippe Aghion, and Gianluca Violante (2001) Deunionization, technical change and inequality, *Carnegie-Rochester Conference Series on Public Policy* 55, 229-264.

[5] Aghion, Philippe and Peter Howitt (1998) *Endogenous growth theory*. Cambridge, MA: MIT Press.

[6] Alesina, Alberto and Edward Glaeser (2004) *Fighting Poverty in the US and Europe: A World of Difference*. Oxford: Oxford University Press.

[7] Allegretto, Sylvia A. (2011) The State of Working America's Wealth. EPI Briefing Paper N. 292, March 23, 2011.

[8] Autor, David, Lawrence Katz, and Alan B. Krueger (1998) Computing Inequality: Have Computers Changed the Labor Market? *Quarterly Journal of Economics* 113, 1169-1214.

[9] Baran, Paul and Paul M. Sweezy (1966) *Monopoly capital*. New York and London: Monthly Review Press.

[10] Barbosa-Filho, Nelson and Lance Taylor (2006) Distributive and demand cycles in the US economy – A structuralist Goodwin model. *Metroeconomica* 57, 389-411.

[11] Barone, Guglielmo and Sauro Mocetti (2016) Intergenerational mobility in the very long run: Florence 1427-2011. Bank of Italy WP 1060, April 2016.

[12] Basu, Deepankar (2009) A simple test of the neoclassical view of production, technical change and distribution. *Metroeconomica* 61, 593-621.

[13] Bénabou, Roland and Jean Tirole (2006) Belief in a Just World and Redistributive Politics. *Quarterly Journal of Economics* 121, 699-746.

[14] Blau, Francine D. and Lawrence M. Kahn (1996) International Differences in Male Wage Inequality: Institutions versus Market Forces. *Journal of Political Economy* 104, 791-836.

[15] Bowles, Sam and Herbert Gintis (1975) The problem with human capital theory – a Marxian critique. *American Economic Review* 65, May, 74-82.

[16] Bowles, Sam and Herbert Gintis (1976) *Schooling in capitalist America*. New York: Basic Books.

[17] Bowles, Sam and Herbert Gintis (2002) Schooling in capitalist America revisited. *Sociology of Education* 75, 1-18.

[18] Borghans, Lex and Andries de Grip (eds.), (2000) *The Overeducated Worker? The Economics of Skill Utilization.*Cheltenham, UK: Edward Elgar.

[19] Card, David and Thomas Lemieux (1996) Wage Dispersion, Returns to Skills, and Black-White Wage Differentials. *Journal of Econometrics* 74, 319-61.

[20] Card, David, Francis Kramarz, and Thomas Lemieux (1999) Changes in the relative structure of wages and employment: a comparison of the United States, Canada, and France. *Canadian Journal of Economics* 32, 843-877.

[21] Carvalho, Laura and Armon Rezai (2016) Personal Income Inequality and Aggregate Demand. *Cambridge Journal of Economics* 40, 491-505.

[22] Duménil, Gérard and Dominique Lévy (1999) Being Keynesian in the short term and classical in the long term. *Manchester School* 67, 684-716.

[23] Duménil, Gérard and Dominique Lévy (2003) Technology and distribution: historical trajectories à la Marx. *Journal of Economic Behavior and Organization* 52, 201-233.

[24] Dutt, Amitava K. (1989) Accumulation, distribution and inflation in a Marxian-Post Keynesian model with a rentier class. *Review of Radical Political Economics* 21, 18-26.

[25] Dutt, Amitava K. (1990) *Growth, distribution and uneven development.* Cambridge, UK: Cambridge University Press.

[26] Dutt, Amitava K. (2005) Globalization, South-North migration and uneven development. In Partha Gangopadhyay and Manas Chatterji (eds.), *Economics of Globalization*, pp.115-129. Aldershot, UK: Ashgate.

[27] Dutt, Amitava K. (2010) Keynesian growth theory in the 21^{st} century. In Philip Arestis and Malcolm Sawyer (eds.), *Twenty-first Century Keynesian Economics*, pp.39-80. Basingstoke: Palgrave Macmillan.

[28] Dutt, Amitava K. (2011) The role of aggregate demand in classical-Marxian models of economic growth. *Cambridge Journal of Economics* 35, 357-82.

[29] Dutt, Amitava K. and Roberto Veneziani (2011) Education, growth and distribution: classical-Marxian economic thought and a simple model. *Cahiers d'économie politique* 61, 157-185.

[30] Dutt, Amitava K. and Roberto Veneziani (in press) Education and human capitalists in a classical-Marxian model of growth and distribution. *Cambridge Journal of Economics*.

[31] Egger, Hartmut and Volker Grossmann (2004) Non cognitive Abilities and Within-Group Wage Inequality. *IZA Discussion Paper* n. 1024.

[32] Flaschel, Peter (2009) *Topics in classical Micro- and Macro-economics*. New York and Berlin: Springer.

[33] Foley, Duncan K. (1986) *Understanding Capital*. Cambridge, MA: Harvard University Press.

[34] Foley, Duncan K. and Thomas R. Michl (1999) *Growth and distribution*. Cambridge, MA: Harvard University Press.

[35] Franzini, Maurizio and Michele Raitano (2014) Wage gaps and human capital: looking for an explanation. Mimeo, University of Rome I.

[36] Freeman, Richard B. and Lawrence F. Katz (eds.), (1996) *Differences and Changes in Wage Structures*. Chicago: University of Chicago Press.

[37] Galor, Oded and Omer Moav (2006) Das Human-Kapital: A Theory of the Demise of the Class Structure. *Review of Economic Studies* 73, 85-117.

[38] Galor, Oded and Daniel Tsiddon (1997) The distribution of capital and economic growth. *Journal of Economic Growth* 2, 93-124

[39] Galor, Oded and Joseph Zeira (1993) Income distribution and macroeconomics. *Review of Economic Studies* 601, 35-52.

[40] Garcìa-Peñalosa, Cecilia and Stephen J. Turnovsky (2015) Income inequality, mobility, and the accumulation of capital. *Macroeconomic Dynamics* 19, 1332-1357.

[41] Giddens, Anthony (1973) *The class structure of the advanced societies*. London: Hutchison and Co.

[42] Glyn, Andrew, Alan Hughes, Alain Lipietz, and Ajit Singh (1990) The rise and fall of the Golden Age. In Steven A. Marglin and Juliet Schor (eds.), *The Golden Age of Capitalism. Reinterpreting the Postwar Experience*, pp.39-125. Oxford: Clarendon Press.

[43] Goodwin, Richard M. (1967) A Growth Cycle. In Charles H. Feinstein (ed.), *Socialism, Capitalism and Economic Growth*, pp.54-58. Cambridge: Cambridge University Press.

[44] Greenstone, Michael, Adam Looney, Jeremy Patashnik, and Muxin Yu (2013) Thirteen Economic Facts about Social Mobility and the Role of Education. Brookings Institution, June 26, 2013.

[45] Harris, Donald J. (1978) *Capital accumulation and distribution*. Stanford, CA: Stanford University Press.

[46] Hirsch, Barry T. and Macpherson, David A. (2003) Union Membership and Coverage Database from the Current Population Survey: Note. *Industrial and Labor Relations Review* 56, 349-54

[47] Jaumotte, Florence and Carolina Osorio Buitron (2015) Inequality and Labor Market Institutions. *IMF Staff Discussion Note*.

[48] Kaldor, Nicholas (1961) Capital accumulation and economic growth. In Friedrich A. Lutz and Douglas C. Hague (eds.), *The theory of capital*, pp. 177-222. New York: St Martin's Press.

[49] Kaldor, Nicholas (1966) Marginal productivity and the macroeconomic theories of distribution. *Review of Economic Studies* 33, 309-319.

[50] Katz, Lawrence F. and Kevin M. Murphy (1992) Changes in Relative Wages, 1963-87: Supply and Demand Factors. *Quarterly Journal of Economics* 107, 35-78.

[51] Krueger, Dirk, Kurt Mitman, and Fabrizio Perri (2016) Macroeconomics and Household Heterogeneity. In John B. Taylor and Harald Uhlig (eds.), *Handbook of Macroeconomics*, pp.843-921. Amsterdam: North Holland.

[52] Lemieux, Thomas (2006) Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill? *American Economic Review* 96, 461-498.

[53] Lucas, Robert E. (1988) On the mechanics of economic development. *Journal of Monetary Economics* 22, 3-42.

[54] Marglin, Stephen A. (1984) *Growth, distribution and prices*. Cambridge, MA: Harvard University Press.

[55] Marx, Karl (1867) *Capital*, Vol. 1. Moscow: Progress Publishers, 1948.

[56] Marx, Karl and Friedrich Engels (1975-2005) *Collected Works*. Moscow: Progress Publishers, London: Lawrence and Wishart, and New York: International Publishers.

[57] McCulloch, John R. (1825) *The Principles of Political Economy*, 5^{th} edition. Edinburgh: Adam and Charles Black, 1864.

[58] Mill, John S. (1848) *Principles of Political Economy with some of their applications to Social Philosophy*. In *Collected Works of John Stuart Mill,* volume 2. Toronto: University of Toronto Press, 1965.

[59] Nakajima, Tetsuya, and Hideki Nakamura (2012) How do elementary and higher education affect human capital accumulation and inequality? A note *Macroeconomic Dynamics*16, 151-158.

[60] Norton, Michael I. and Dan Ariely (2011) Building a better America - One wealth quintile at a time. *Perspectives on Psychological Science* 6, 9-12,

[61] Nussbaum, Martha C. (2010) *Not for profit! Why democracy* needs *the humanities*. Princeton, NJ: Princeton University Press.

[62] O'Brien, Denis P. (1975) *The classical economists*. Oxford: Clarendon Press.

[63] O'Donnell, Margaret G. (1985) *The educational thought of the classical political economists*. Lanham, New York and London: University Press of America.

[64] Onaran, Ozlem, Engelbert Stockhammer, and Lucas Grafl (2011) Financialisation, income distribution and aggregate demand in the USA. *Cambridge Journal of Economics*35, 637-661.

[65] Pasinetti, Luigi L. (1960) A mathematical formulation of the Ricardian system. *Review of Economic Studies* 27, 78-98.

[66] Petith, Howard (2008) Land, technical progress and the falling rate of profit. *Journal of Economic Behavior and Organization* 66, 687-702.

[67] Piketty, Thomas (2014) *Capital in the 21st Century*. Cambridge, MA: Harvard University Press.

[68] Romer, Paul M. (1990) Endogenous Technological Change. *Journal of Political Economy* 98, S71-S102.

[69] Skott, Peter and Fabian Slonimczyk (2012) Employment and Distribution Effects of the Minimum Wage. *Journal of Economic Behavior and Organization* 84, 245-264.

[70] Smith, Adam (1776) *An Inquiry into the Nature and Causes of the Wealth of Nations*, Volume 1. In Roy H. Campbell and Andrew S. Skinner (eds.), *The Glasgow Edition of the Works and Correspondence of Adam Smith,* Vol. 2. Oxford: Oxford University Press, 1976.

[71] Stokey, Nancy J. (1991) Human capital, product quality and growth. *Quarterly Journal of Economics* 106, 587-616.

[72] Tavani, Daniele (2013) Bargaining over productivity and wages when technical change is induced: implications for growth, distribution, and employment. *Journal of Economics* 109, 207-244.

[73] Tavani, Daniele and Ramaa Vasudevan (2014) Capitalists, workers, and managers: Wage inequality and effective demand. *Structural Change and Economic Dynamics* 30, 120-131.

[74] Unger, Roberto M. (2007) *Free trade re-imagined*. Princeton, NJ: Princeton University Press.

[75] Uzawa, Hirofumi (1965) Optimum technical change in an aggregative model of growth. *International Economic Review* 6, 18-31.

[76] Wood, Adrian (1994) *North-South Trade, Employment and Inequality*. Oxford: Oxford University Press.